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MEN AND INDUSTRY.*

BY

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The hall in which I have the privilege of speaking tonight has been honoured by the presence of many who have written their names high in the history of science and the arts. The Franklin Institute has been distinguished not only for the fine educational opportunities which it offers, but also for the advancement of science in all of its forms. It is therefore with considerable temerity that I find myself speaking in surroundings which have been honoured by many others whose profundity of knowledge leaves me indeed an insolvent competitor.

However great has been the advancement of scientific learning in the last few decades, we are today confronted with social and economic problems of even greater importance to those nations which claim both culture and civilization. Therefore, moved by these considerations, I venture to present to you my remarks in the three following aspects—

First, in a discussion of what I hope may impress you as an enlightened view of the position of manpower in industry as distinguished from materials;

Secondly, to outline the influence of modern industry on international relations; and

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Thirdly, to point out an historical analogy which may well sound a warning to these peoples and nations who now regard themselves as the custodians of civilization and culture.

To provide the necessary perspective for men and materials in industry, let us consider for a moment the historical background in the relations between what was, in older days, the master and the servant, but, in more modern times, has become the employer and the employee: it is the history of the evolutional improvement of the masses, the battle against a servitude as natural as it was deplorable, and the progressive advancement of labour in independence, dignity and wisdom.

In ancient days the world was divided broadly into two classes—those who ruled and those who served—the master and the slave. Mechanical devices, excepting in their simpler forms were unknown, and, in its mechanical sense, energy was chiefly found in the backs of men. This was of necessity a condition of antiquity. Culture, knowledge and power were for but few, and were the right of heredity or the reward of the conqueror and those of his train. The value of men was estimated by physical output and their ability to resist the ravages of ceaseless toil. Life indeed represented a precarious and arduous existence. Ignorance of better conditions induced an amazing satisfaction. The greatest of all pursuits in those ancient days was warfare. Wars were waged at will for the benefit of the few, and the uncomplaining masses regarded their sacrifice upon the altar of Mars as a necessity of life. It was the day of the master and the slave.

The first dawn of the dignity of industry as distinguished from the glory of military pursuit appeared in the formation of craft guilds which came in time to flourish in some countries and awakened the pride of the master craftsman and the artisan. These guilds grew in power and importance, and contributed much to the advancement of industrial pursuits. In most instances the home of the master craftsman was likewise the factory. There he gathered his apprentices, taught his trade, and exercised a paternal influence. The master craftsman, his family and apprentices, became partly a family and partly an industrial unit. The remains of these old guilds are found today notably in England and are represented in

modern times by the great city companies such as the Ironmongers, the Fishmongers, the Goldsmiths and Silversmiths companies, and many others. Although these great London city companies have largely ceased to have any industrial importance, they still exercise a benevolent and worthy influence, and cherish the best traditions of the craft.

The development of steam as a form of energy, and the invention of machines which performed mechanically the work of many hands, completely changed the social and economic structure of industrial and commercial nations. The factory displaced the shop of the master craftsman, and the artisan became a servant of capital. These combinations were a necessary evolution in the progress of industry. It was found that the establishment of factories with steam driven machines involved a greater capital than was within reach of the former master craftsman; consequently companies were formed for this purpose, and numbers of individuals acquired interests in these enterprises through financial participation. As a result, the paternal influence of the master craftsman disappeared and those who toiled found themselves working for enterprises whose natural object was the production of satisfactory financial returns to the shareholder. This applied equally to mines, railways, textile and steel works, and finally to most forms of industrial activities. In the initial stages of this development the employing company, in its relations to its hands, quite necessarily acted as a unit. The employees attempted to act as individuals. Remembering that in those days there still lingered, in the minds of those who ruled industrially, the flavor of feudal days and the traditions of master and servant, oppression displaced the benevolent influence of the master craftsman, and conditions in most industrial centres became intolerable to the worker. As a protective measure, the inevitable evolution was the combination of workers into craft unions, and they then effected that unity of purpose and action which characterized the employer. Naturally, in such an atmosphere, conflict was inevitable,—a conflict which was stupid, destructive and illogical.

After some decades of such conditions there arose sane minds on both sides who recognized that industrial warfare was just as costly and quite as unsatisfactory as the military

variety, and a more intelligent policy made its appearance which was characterized by a recognition on both sides of the rights of the other, until we have today, happily, in many important fields a general feeling of consideration between capital and labor and a recognition of each other's rights. Such, briefly, has been the arduous and slow advancement of the toiler from slave to his present position.

The progress is by no means finished, indeed nothing in this world is ever finished. There is still another step to be made, and now we stand upon the threshold of a new and better relationship which I describe as "the partners." The future contact between capital and labor and between the employer and the employee will, in the decades to come, find its greatest progress and its finest expressions in the era of The Partners.

Industrial pursuits are fundamentally the utilization by efficient methods of men and materials. The steel rail, the bridge, the building, the locomotive, the automobile, and all of those appliances with which we are familiar, represent the application of men and materials; materials in their raw form wrought into some useful device by brain and manual effort, but the inanimate material is of lesser importance than the human contribution which turns those materials to useful purposes. We investigate, analyze and test the materials we purchase; we store, protect, and treat them with something akin to paternal care. Those whose brains and muscles revolve the wheels of industry, converting bricks into buildings and steel into structures, merit equal care. The skilled mechanic, the trained employee, is commercially worth quite as much as many units of material. When, for one cause or another, he leaves the service of his employer, there is lost knowledge, experience, and productivity. These are the things which enlightened employers cherish and protect.

It is obvious that capital as applied to production becomes quite useless unless accompanied by labor. Both are equally essential for economical and efficient production. One cannot proceed far without the other. Dissatisfaction on the part of one or the other interferes with the smooth running of the whole. In the true sense of the word, these two great elements must be partners if the interests of each are to be served.

No industrial or commercial enterprise, no matter how scientific its formulae or how efficient its administration, can succeed unless there is contentment on the part of workers and a desire born of pride to give of their best. No general, however great his skill, ever won victories with a mutinous army. Labor merits its right to partnership, and sagacious capital will welcome such a partnership. With a recognition of the partnership principle, there automatically must disappear that contention and strife which is just as expensive in the industrial world as it is in the relations between nations. Warfare has been described as the only game in which both sides lose. Therefore, why provoke disaster? In the present day, and in the days to come, those industries will prosper most which accept labor as an equal partner, and labor will best prosper when it recognizes the obligations and the dignity of that partnership.

The power of both labor and capital is recognized, but great power implies equally great responsibility. To those who are given the ability and opportunity to create there is also the obligation to build. To neglect that obligation is both cowardly and stupid. Enlightened thought and experience teach that labor and capital together will make their greatest contribution to the welfare of mankind in the active and unceasing promotion of the partnership theory in all forms of industrial activity. This doctrine, carried to its logical conclusion, means to the employee, contentment, happiness, continuity of work and pride in his undertaking. It brings to the employer industrial tranquillity and freedom from anxiety, with progressively improving results: to the world at large it constitutes an example of what can be accomplished by the development of a sound theory of co-operation consistently pursued.

The practical method establishing the partnership to which I refer must, to some extent, vary with the character of the industrial activity involved, but there are certain principles and objectives which are relatively simple. The primary object is the excitement of interest in the daily task on the part of the workman. To accomplish this, his suggestions for improvement should be solicited and, where useful, adopted. He must also be allowed, through his representatives, that

voice in management consistent with the responsibilities of the proprietor. The contentions which spring from the maintenance of discipline are negligible, because, in my experience, with the advent of the partnership plan, infractions almost entirely disappear. In short, labor should be recognized as a partner with all of the privileges of suggestion and advice which are appropriate to a partner. If the inauguration of the theory brings increased output or lower unit costs, management should promptly recognize the debt in some appropriate and tangible form. For example, the results from the introduction of the cooperative or partnership movement in the shops of the Canadian National Railways have been so gratifying that all shop crafts have been given voluntarily a week's holiday each year on pay.

The haunting fear of the workman is dismissal, or, from one cause or another, the discontinuance of his livelihood. Not many workmen are able to save against evil times, and when those times come they bring misery and distress to those whose only capital is skill of hand. An active and friendly collaboration between management and men offers an opportunity to increase greatly continuity of employment. Briefly put, the partnership plan offers to both employer and employee—

(1) Improved continuity of employment to the worker;
(2) A more kindly and satisfactory relationship between employer and employee;
(3) An improvement in output with reduced unit costs;
(4) The mobilization of the brains of workmen in support of management and the provision of an outlet for their ingenuity;
(5) A more accurate conception on the part of both capital and labor of each other's rights and responsibilities.

I will now ask you to consider with me the second topic of my lecture, namely the influence of modern industry in international relations. World affairs have become so interwoven that no thoughtful person can ignore the activities which but a few decades ago would affect at most the nation but today may well provoke international disputes.

Until relatively modern times, matters relating to indus-

trial operations were of minor importance in the consideration of international relations. In past centuries international relations turned upon religious contentions, the ambitions and prejudices of rulers, and last, but by no means least, the predatory instincts of nations. Now and then the demands of trade provoked warfare, but such causes usually developed from situations affecting food supplies. Religious considerations and predatory instincts were the larger factors affecting international conflicts.

It is only since the disaster of the last great war that the futility, waste and tragedy of warfare have appeared in proper perspective. Only within the last few years has there been an organized movement for world-wide peace and the provision of international machinery for the attempted prevention of war. No theory can become the policy of any nation until the majority of the electorate is sufficiently impressed to demand adoption, for numerous examples demonstrate that no law can be successfully enforced in the absence of considerable public approval. Peace amongst nations will only appear when a majority in each nation, and sufficient nations themselves, are inspired by reason to seek such an objective. This, somewhat differently put, means that the structure of amiable international relations must be built upon a foundation of insistent public opinion. In our present wide-spread forms of popular government, the statesman lends an attentive ear to the murmurings of the masses, for it is by their mandate that he holds his place.

I have previously explained that in bygone centuries, military affairs, statecraft and religion—and the three were often synonymous—represented the only worthwhile activity of great minds: but in modern times the creation and harnessing of cheap energy has replaced the backs of slaves and given to industry that unlimited scope which finds its expression in the great enterprises of modern times. Happily the toiler has been freed, and energy enslaved. To this simple fact may be attributed the enormous social and economic changes which have occurred in the last century, and today we find the greatest field for human endeavour in the occupational preëminence of finance and industry. The captains of industry have replaced the marshals of armies. Religious

contentions, the predatory desires of monarchs, the thirst for revenge (all fruitful disturbances in international relations) have largely disappeared, and with the advance of finance, commerce and trade as preponderating influences in the world today there is found the most potential cause for strife and conflict. Psychologically the underlying motive is competition, expansion and the penetration of international markets. The lure of competition and ambition inevitably leads to commercial invasions and the adventures of world wide trade. But as the armies of trade march across nations they immediately come into conflict with the barriers of tariffs and national pride, which, though intangible, are just as distinctly fortifications to be conquered and demolished as the more tangible military works which were encountered by the captains of ancient invading armies. The development of modern industry and improved international relations must go hand in hand if either is to achieve potential results. Even today the effect of tariff barriers and commercial invasions upon international relations are not wanting. I have no opinion to express nor criticism to make upon the commercial and trade policies of any nation, but he is a poor observer who does not at least recognize and assess their effect.

Modern industry has become so highly organized, its efficiency and volume of production so largely increased through the introduction of improved methods and machinery, that great industrial nations must seek ever widening and far-flung markets. Cheap and effective transport has aided the producer in reaching markets which were previously closed. In this struggle, which is both offensive and defensive, lies unquestionably the real menace to world peace. It is the field in which modern industry and its leaders can play their finest part.

He is a courageous man who attempts advice to a nation, and it is a dangerous performance if he happens to be a visitor from abroad. In what follows, my only excuse must rest upon the fact that frank speech, which I hope finds its source in sincerity and is clothed with courtesy, can offer no offence. The annals of history disclose nothing comparable to the romantic, tremendous development of the United States. But 154 years ago there was born on the eastern shores of the

North American Continent a new nation which asserted its independence from European control and boldly strode forth in its newly found independence. Many were its infant problems, painful were its youthful maladies, but with the lusty strength and determination of youth it proceeded upon its growth to national manhood. At one stage in its career it suffered an internal dissension which approached disaster. Aloof largely from the old world, it pursued its destiny, and today the United States stands foremost in opulence, power, and influence. But these attributes likewise inevitably carry certain responsibilities, and one of those responsibilities must be to give of its best in world leadership. Measurably this responsibility is being fulfilled. The counsel of great American bankers is sought in the settlement of international financial affairs; nations listen with attention to the comments of her statesmen; but there still lingers in the minds of the great mass of the population of this country the echo of what is alleged to be the advice of the Father of his Country to beware of foreign entanglements. Just what Washington meant by this may be arguable, but had he suspected the tremendous development of his country, its vast power and wealth, I think he would have been one of the first likewise to recognize the responsibilities of his nation in its relationship to other and less fortunate countries.

The United States lives in relative physical isolation from foreign countries. Its inhabitants as a whole do not come into that intimate contact with other nations which proximity permits in Europe. The people of this land have grown and developed largely in a soil of their own, and, springing from that soil, have found their own psychology and developed their particular national characteristics. Being a peaceful nation and growing largely without foreign influence, it is but natural that foreign statescraft should be regarded as the effort of one country to gain an advantage over the other, and the average individual looks upon Europe as a collection of squabbling nations actuated chiefly by selfish desires. Let us examine the lesson of history. Since the disintegration of the Roman Empire and the removal of that powerful and restraining hand, coupled with the invasion of barbaric tribes from the East, Europe has been pretty much in continuous warfare.

For centuries it was a melting pot, bubbling over here and there with warfare and intrigue. These conditions existed through the long march of years in which international contention and warfare were considered thoroughly respectable and admirable pursuits. Substantially every nation of Europe has suffered invasion and borne the agonies of war. On the North American Continent, although the United States has fought one internal war and engaged in four foreign conflicts, the popular point of view and psychology of its people with respect to warfare is quite different from that which exists abroad, and if this great country with its opportunities for leadership is to make effective contribution to the advancement of civilization, its people must determine their opinion of other nations with due regard to the history and environment of those nations; in other words, intelligent examination and tolerance must be invoked.

Enlightened industry is solving the difficult problems which affect the relationship between capital and labour. Strikes are the exception rather than the rule. A fine recognition of the responsibilities of both capital and labour has introduced the coöperative and partnership theory to which reference has been previously made. Both the contending factors have found that they advance the more through coöperation; and if this smaller, but none the less complicated problem, is at least approaching a solution through the efforts of modern industry and its captains, may not that furnish an example which brings hope to those who would solve the larger problem of improved international relations?

The first factor in improved international relations and to which all others are subordinate, must be an understanding of each people with respect to other peoples. That is the first point of attack, and until some measurable form of progress is so made, efforts looking to the improvement of international relations must fail. The problem is largely one of education. Education embraces many forms. Principally it is achieved by study, travel, and instruction from those who are competent to teach. No individual or group of individuals can understand or maintain amiable contacts with others of their kind without a knowledge of the history of those with whom they are dealing, together with the causes and influences of the

great movements of history. An enterprise is more than bricks, mortar and machinery; its vitality is the kind of spirit which moves it. A nation is more than fields, workshops and mines; its reactions will be determined by the spirit and mentality of its citizens. And if we are to understand the peoples of other nations, we must understand the spirit that moves within them, that living spark, intangible and ethereal, which is the self-starter of all national activities. There is an old proverb "KNOW THYSELF," but far more important it is to know thy neighbour. The spearpoint of improved international relations must be the education of races in the history of those nations with whom they come into commercial contact. By history I do not mean a recital of politics and warfare and dates; I mean a knowledge of those elemental things which, operating through ages past, have produced the Englishman, the Frenchman, the German, and so on. Let me give you a simple illustration of what I mean. Those of you who have visited England and travelled through the towns and countryside, have perhaps observed that every house with its surrounding gardens is shielded from public view; high walls, or hedges lend to the home complete privacy, and behind this barrier the householder enjoys, with his family, the beauties of his garden. The American traveller along the highway remarks "What a selfish man is this Englishman! Doubtless behind his walls there is a garden which might delight the eye and bring pleasure to the passerby, and yet he selfishly keeps this to himself. How different it is in our country where one lawn and garden merges into the other, where the proprietor in the pride of his flowers, his shrubs and fountains, rejoices to share them with the stranger. Indeed, this Englishman is a selfish and surly fellow." But back of all that there is a historical reason for the difference. In the old feudal days, which you are aware represented more than one century in England and in continental countries, the stronghold of the local baron represented the only refuge in times of trouble, and trouble was the rule, not the exception. At night the herds were driven within the castle walls by the retainers of the lord, the drawbridge was raised, the portcullis lowered and guards mounted. Every precaution was taken to prevent the expected attack. Many farm houses were surrounded by moats, and at night each unit retired within itself

for protection against a neighboring predatory baron or bandit. And this condition continued so long that it eventually produced that definite psychological effect which in modern times finds itself reflected in enclosed gardens and the withdrawal of the family within itself. On the North American Continent circumstances were exactly the opposite. Those who originally came to these shores were banded together to resist the attacks of savages and the ravages of wild beasts. The traveller along the highway, instead of being a spy, was a welcome guest and often brought with him the only news of the outside world. In the Province of Quebec, for example, we find the farms are long and narrow, radiating from villages in which the inhabitants were gathered to more successfully resist the attacks of the Indians. So we find that on this continent the effect of environment was to drive people together in the promotion of communal life and intercourse, and hence our gardens and grounds are exposed to public view and the entire physical atmosphere is one of complete inter-communication. Thus you see, from one simple illustration, how history and and environment create certain characteristics which are often misunderstood by the visitor.

The late war was epochal in more than one way, but chiefly because it has drawn most of the nations of the earth together in what seems to be a somewhat contentious attempt to bind up their economic wounds, and, even though present contacts may be contentious, at least there is a more intimate contact than existed in the past. Out of that contact there will probably grow some better understanding of each other's difficulties. However, disastrous the Great War may have been, it certainly impressed upon the minds of all intelligent beings that warfare is to be prevented at all hazards and that no nation can maintain itself in complete isolation. Business frontiers have been widely extended, modern industry itself is world-wide and interwoven, business men travel over the Seven Seas and pursue international contacts to a greater extent than ever before. When international relations become strained or difficult, industry and the business man suffer the most. He has the bigger stake in the maintenance of amiable relations, and if he has more to lose then he should be the first to initiate an organized effort for the maintenance of those conditions which will prosper him the greatest.

When all is said and done, if we are to promote amicable international relations we must begin to build from the bottom up, first planting the good seed in the minds of the individual and groups of individuals who in coöperative combination determine national policies. If the result is to be achieved it must be through education and tolerant contacts. Modern industry will make its greatest contribution to this end by first setting a fine example with respect to those who serve within its own walls and the stranger beyond its gates but more importantly by a determined and organized effort to promote happier international relations.

The history of ancient days reveals the rise and fall of various succeeding civilizations, each rather improving upon its predecessor until we find the Roman Empire which gathered under its protecting and disciplinary wing substantially all of the known civilization and culture of the period in which that Empire reached the maximum of its power and influence. There may have been, in far off Cathay, other civilizations but to all intents and purposes the statement is not far wrong that at that epoch the Roman Empire was the custodian of culture and order. Now the progressive allurements of civilization are difficult of resistance, and so it came about that Rome, enervated and demoralized, yielded to the invasion of barbarian nations who had been knocking at her frontiers for some centuries, and finally, in the march of years, the empire was broken into a number of warring fragments. In short, the custodian of learning, knowledge and all of the things which in that day were called "culture" disappeared. Civilization slipped into what are known as the "dark ages," and only with the revival of learning known as the Renaissance did the world's progress again resume its arduous climb to higher cultural altitudes. The story of the decline, stagnation, and resumed progress of civilization might easily fill many volumes, but what I have said sufficiently paints the picture for my purpose.

I think it may be regarded as substantially a fact that prior to the Great War, European and North American nations collectively, insofar as the trusteeship of learning and culture is concerned, may measurably be regarded as in the same position as the Roman Empire at the height of its power. To be sure, there are certain Oriental or Eastern cultures and

influences which embrace philosophies and schemes of life which are attractive to many, and I do not mean to arrogate to Caucasian civilization all that is good in the world, but broadly speaking Europe and the North American Continent in power, influence, and learning, may be regarded as the leaders of civilization.

The late tragedy of the great war which has exceeded every other catastrophe in evil results, has taken a heavy toll in blood, brains, wealth and influence from European nations. Some of those nations, which prior to the war, represented responsible empires, are now irresponsible fragments. The ravages of battle have robbed the Caucasian race in this generation of the flower of its youth. That, in itself, represents a tremendous liability. The restraining hand of world discipline is weakening. Already serious influences, of a magnitude not to be despised, seek the overthrow of all those finer traditions to which we owe allegiance and hold to be sound. I have no quarrel with advanced theories or doctrines, other than to make the incontrovertible statement that such theories and doctrines should be accepted only after full trial and experience and then by the orderly process of evolution rather than by the destructive method of revolution. Moreover, each nation must be allowed to develop its own scheme of government and society, without interference from abroad. The East appears to be in a general state of disruption and disorder, some sections struggling with neighboring sections for supremacy. With all of this, there is a weakening of orderly Caucasian control and the awakening of inexperienced and uncultured hordes. In short, the analogy seems striking, and it may be that the barbarian is today knocking at the frontiers of European civilization, and, in the invasion, should it come, culture and civilization may perhaps slip again into an era of dark ages to resume in some subsequent century its painful climb toward Olympus.

The dissemination of knowledge and the acquisition of learning has no value unless it brings in its train tolerance of thought and considered action. Civilization's great protection against the invasion of the barbarian and the bloodthirst of the revolutionist is that accuracy of thought and orderly procedure which will allow the peoples of the world to work out their destinies without undue pain and distress.

AN INVESTIGATION OF THE FUNDAMENTAL CRITICAL SPEEDS OF MULTIPLE SHAFTS.*

BY

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I. INTRODUCTION.

A. Purpose of Analysis.

The material presented in the following pages is a part of a theoretical investigation into the fundamental critical speeds of multiple shafts conducted in the research and development sections of a manufacturer of electrical power apparatus. It represents a resumé of much of the thought which has previously been expressed in scientific publications and design manuals upon the problems encountered in the construction of large rotating machines. To this fundamental knowledge little may be added, but it may be interpreted in a more practical manner, and re-arranged to meet conditions as they are visualized at the present time.

It is the purpose of this analysis to present in definite form, by two methods, all the explanations, equations, and references necessary for a general understanding of the calculation of the critical speeds of multiple shafts. By the term "multiple shaft" is meant a continuous shaft formed by the alignment of the rotating elements of several machines, in which the unit shafts are rigidly coupled together. The critical speeds of such a structure are influenced by the mechanical characteristics of each of the individual units, and the solution is found by the analytical combination of these elements.

B. General Statement of Problem.

It is a well established fact that the rotating elements of machinery at certain speeds become dynamically unstable, allowing large impulses of energy to be transferred from the

* Submitted to the Graduate School of the University of Pittsburgh in partial fulfilment of the requirements for the degree of Master of Science.

moving to the stationary parts. The phenomena of the disturbance experienced occur at the critical speed of the structure. In such a condition it has been found[1] that the path traversed by the center of mass on the shaft describes a certain orbit about its true axis. This action may be aggravated by an unbalance or other physical imperfection, but it is not dependent upon it. Periodic forces are thereby impressed upon the bearing supports of the shaft and are transmitted to the foundation. In the resonance condition, these forces tend to become continually greater: sometimes causing destruction before a point of equilibrium is reached. On each side of this critical speed is a range in which the vibration is objectionable for smooth operation. Also, it has been observed that critical vibration may occur at one-half or one-third the fundamental critical speed in multiple units, with consequent effects upon performance. It is essential, therefore, that designers should be aware of the behavior of their machines in order to avoid critical ranges of vibration within the operating speeds.

The problem of finding the several critical speeds, where several machines are rigidly coupled together, forming a multiple shaft condition, is more difficult than that of determining the single critical speed of an individual machine. In the general solution, the individual units are first investigated to ascertain certain structural factors; the constants thus obtained are then combined in analytical functions of a given type; and the solution of these functions determines the critical frequencies to which the mechanical system is susceptible.

In this paper, after a short theoretical review, is given the development of the solutions for the critical speeds of multiple shafts by two methods, with both rigid and flexible bearing supports, followed by the application of the methods to several actual machines.

C. Theoretical Methods of Solution.

A rotor, defined as the rotating element in an electrical machine, and consisting of a built-up structure supported upon a shaft, is subject to natural vibration with various modes of

[1] Stodola, A., "Steam and Gas Turbines" (See Bibliography), Art. 90, p. 437.

motion. We are interested in investigating the rotor conditions which determine the frequencies at which this natural vibration will occur, and the resultant types of motion.

1. *Systems with One Degree of Freedom.*

When the load on a shaft can be considered as a concentrated mass, as in many practical cases, the problem involved in the critical speeds of single shafts is equivalent [2] to the problem of vibration in elastic systems having one degree of freedom. In general, this elastic system consists of a heavy mass supported by a flexible member in such a manner that the sudden release of a force applied to the mass would set it in to vibratory motion. A system of this type is commonly represented in mechanics by a weight on a spring. From the elementary laws of mechanics, the free motion of the weight, with the effects of damping neglected, is determined by the following differential equation: [3]

$$M\frac{d^2y}{dt^2} + Ky = 0. \tag{1}$$

In Fig. 1 is shown a spring, with rigidity costant K, supporting a weight of mass M, which weight is displaced and

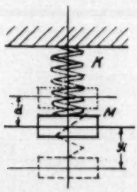


FIGURE 1.

held at rest by an applied force. If, at the moment the force is released, the time $t = 0$, and the weight M has a displacement y_i from the position of equilibrium, the solution for the motion of the weight, from Equation 1, will be

$$y = y_i \cos\sqrt{\frac{K}{M}}t, \tag{2}$$

[2] Lord Rayleigh, "Theory of Sound" (See Bibliography), Vol. I, pp. 111 and 287.

[3] Timoshenko, S. and Lessells, J. M., "Applied Elasticity (See Bibliography), Chap. XI, p. 313.

which defines the position of the weight at any instant. The period of the motion, from Equation 2, is found to be

$$p = 2\pi\sqrt{\frac{M}{K}}, \tag{3}$$

and the frequency, the number of vibrations per second, is given as

$$n = \frac{1}{p} = \frac{1}{2\pi}\sqrt{\frac{K}{M}}. \tag{4}$$

The term $\frac{M}{K}$, or $\frac{W}{gK}$, proportional to the static deflection, is generally recognized as the transposition factor in its relation between the statical and dynamical problem. It is particularly useful in shaft calculations of deflection and critical speed.

a. Rigid Bearing Supports.—Referring to Equation 3, it is possible at once to calculate the period of the lateral vibrations of a shaft on rigid pedestals carrying one disc only. The diagrammatic representation of such a shaft with a single load is given in Fig. 2. The natural period of vibration will be

$$p = 2\pi\sqrt{\frac{d}{g}}, \tag{5}$$

where $d\left(=\frac{W}{K}\right)$ is the deflection of the shaft under the

FIGURE 2.

weight of the disc. If the time for one revolution of the shaft is equal to this period, the phenomena of critical disturbance occur.

The angular velocity for critical vibration is noted as

$$\omega = \sqrt{\frac{K}{M}} \tag{6}$$

in radians per second, and the critical speed is observed to be[4]

$$N_c = \frac{60\omega}{2\pi} = 187.7\sqrt{\frac{K}{W}} \qquad (7)$$

in revolutions per minute.

b. Flexible Bearing Supports.—If the shaft is supported upon flexible pedestals, and the flexibility is considered only in the lateral direction, in the case of a single span for the fundamental mode of motion, Fig. 3, the system is again observed

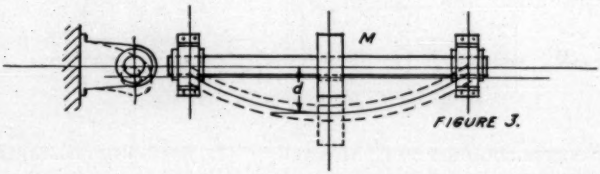


FIGURE 3.

to have one degree of freedom. The same methods of solution may be applied as previously given; it being noted, however, that in the calculation of the static displacement of the weight, the additional deflection due to the flexibility of the bearing supports must be added.

2. *Systems with Several Degrees of Freedom.*

When several individual shafts are rigidly coupled together to form a continuous shaft, the equations of motion are again strictly defined. The problem is then one for a system having several degrees of freedom.

a. Rigid Bearing Supports.—As the simplest example of such a system, a shaft having three supports will be analyzed. It is given that the coupled shafts, shown in Fig. 4, are loaded with a single disc on each span. The static problem involves the necessity of finding the deflection of each shaft at the position of the loads for unit weights placed first on one span and then on the other. The total deflection under the load P_1 will be dependent upon P_2 in such a relation as follows:

$$y_1 = a_{11} P_1 + a_{12} P_2, \qquad (8)$$

[4] Stodola, A., "Dampf- und Gasturbinen," Berlin, 1924, pp. 357–398.

where a_{11} and a_{12} are flexibility constants depending upon the size of the shafts and the location of the supports and the loads. The above relation is a particular case of the Reciprocal

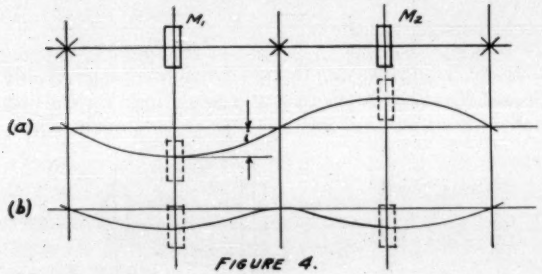


FIGURE 4.

Theorem obtained by C. Maxwell.[5] The flexibility constants are commonly called 'influence factors,' and they are determined from an analysis of the shaft structure by statically indeterminate methods. The values are derived by analytical means when the shafts are uniform in section with concentrated loads, and by graphical constructions when the shafts are variable in section with distributed loads.

In the same manner, for the other span, the deflection is expressed by Equation 9:

$$y_2 = a_{21} P_1 + a_{22} P_2. \qquad (9)$$

Returning to the dynamical problem, the static loads P_1 and P_2 are replaced by equivalent inertia forces corresponding to the moving masses M_1 and M_2 of the vibrating discs. These inertia forces may be expressed in the following form;[6]

$$P_1 \equiv - M_1 \frac{d^2 y_1}{dt^2} \qquad \text{and} \qquad P_2 \equiv - M_2 \frac{d^2 y_2}{dt^2}, \qquad (10)$$

and when these forces are substituted in Equations 8 and 9 for

[5] Maxwell, C., "On the Reciprocal Theorem," *Phil. Mag.*, Vol. XXVII, 1864, p. 294.

[6] Borowicz, "Beträge zur Berechnung Krit. Geschwindigkeiten zwei und merhfach gelagerter Wellen," Dissertation. Munchen, 1915.

P_1 and P_2, the Differential Equations 11 will be obtained;

$$y_1 = -a_{11} M_1 \frac{d^2y_1}{dt^2} - a_{12} M_2 \frac{d^2y_2}{dt^2},$$
$$y_2 = -a_{21} M_1 \frac{d^2y_1}{dt^2} - a_{22} M_2 \frac{d^2y_2}{dt^2}. \qquad (11)$$

These are the equations of the system having two degrees of freedom, the motion being constrained to the y plane. The solutions of Equations 11 are obtained by assuming that

$$y_1 = i_1 \cos \omega t \qquad \text{and} \qquad y_2 = i_2 \cos \omega t, \qquad (12)$$

and differentiating—

$$i_1 = a_{11} M_1 \omega^2 i_1 + a_{12} M_2 \omega^2 i_2,$$
$$i_2 = a_{21} M_1 \omega^2 i_1 + a_{22} M_2 \omega^2 i_2.$$

These equations, when resolved into determinantal form follow:

$$(a_{11} M_1 \omega^2 - 1) i_1 + a_{12} M_2 \omega^2 i_2 = 0,$$
$$a_{21} M_1 \omega^2 i_1 + (a_{22} M_2 \omega^2 - 1) i_2 = 0. \qquad (14)$$

It is seen that solutions for the factors i_1 and i_2 exist only when the determinant in the denominator is equal to zero. The denominator then assumes the form

$$(a_{11} M_1 \omega^2 - 1)(a_{22} M_2 \omega^2 - 1) - (a_{21} M_1 \omega^2)(a_{12} M_2 \omega^2) = 0, \qquad (15)$$

which, when solved in the usual manner, determines the value of the two roots, ω_1^2, and ω_2^2.

Since the above equation is of the second degree, the two roots denote the angular velocities for two different modes of natural vibration. Substituting the lowest root in Equations 13, i_1 and i_2 will have opposite signs, and the shape of the curve obtained will be as shown in Fig. 4 (*a*), which is the fundamental mode of motion. Then, substituting the other root in those equations, i_1 and i_2 will have the same sign, and the shape of the curve will be as in (*b*), which is the second mode of motion. The method sketched here is for the case of a continuous shaft with three supports, and it may be extended directly to any number of supports.

b. Flexible Bearing Supports.—For the case of a continuous shaft with flexible supports, a complete treatment will be given in the Main Discussion, under headings $A(2)$ and $B(2)$, and in the Appendix, of this paper.

II. MAIN DISCUSSION.

In the introduction to this paper, the *use* of influence factors in the solution for the critical speeds of multiple shaft systems has been established by a review of the principles of the Reciprocal Theorem as developed by Maxwell. It is the aim of the following discussion to describe the *derivation* of the influence factors which are required for the completion of the specific problem mentioned above.

Two methods of approach are given, with each method branching to define the influence factors for two conditions of the dyamic system. Solutions for the first condition were originally derived by Maxwell [5] and Borowicz,[6] respectively, and those for the second condition were an outgrowth and continuation of the first. The first condition, as above, deals with the shaft structure alone, considering the shaft to be supported at definite, inflexible points. Since the bearings, the pedestals, and the foundation which support the shaft are not objects of infinite rigidity, and, indeed, in many cases even approach a flexibility commensurate with that of the shaft, it is important that due consideration be taken of the additional freedom which a shaft may enjoy. The second condition, therefore, deals with the shaft structure as carried upon the semi-rigid supports common to practical design.

On account of the scope of the resulting derivations, much of the detailed analysis is to be found in the Appendix in conjunction with examples of the *application* of this development to actual machines.

A. Solution by the Theorem of Moments.

In the derivation of the influence factors of multiple shaft systems, the first method of analysis given is by the Theorem of Moments, a specific solution for statically indeterminate problems. This theorem is based upon the isolation of the individual spans of a structure, and the superposition of forces and moments to create an identical deformation. The

analysis obtained under the above conditions determines the expressions for the influence factors. These factors are then substituted into the reciprocal equations of Maxwell to obtain the equations for the critical speeds. In the following solution, the two conditions of the problem, as previously stated, are set forth; and each condition is analyzed with regard to the case of a continuous shaft with two spans, and the general case of a shaft with a number of spans.

1. *Consideration of Rigid Bearing Supports.*

By the term "rigid bearing supports," it is implied that the points of support of a continuous shaft are rigidly fixed in space. It is assumed, however, that the bearings are free to pivot in the supporting pedestals. The analysis of the actions of the shaft structure under these conditions follows.[7]

a. Derivation of Equations for a Shaft with Three Supports.—If two single shafts, each carrying a load, are rigidly coupled together, as illustrated in Fig. 5, it is necessary to find the

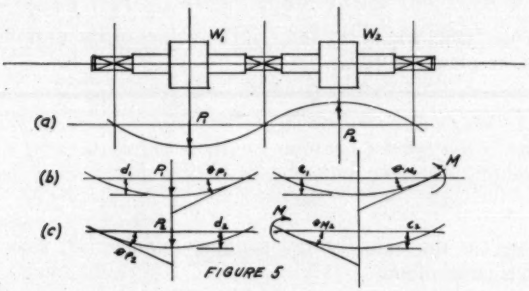


FIGURE 5

deflection of each span due to a unit load applied first on one and then on the other. Since a positive force on one span produces a negative deflection on the other, it is required that the loads on the two spans act in opposite directions.[8] This

[7] Timoshenko, S., "Vibration Problems in Engineering (See Bibliography), Chap. III, p. 159.

[8] Soderberg, C. R., "The Vibration Problem in Engineering," *The Electric Journal*, Vol. XXI, July 1924, pp. 330–334.

condition is shown in Fig. 5 (*a*), where the shaft loadings, P_1 and P_2, are reversed in accordance with the above requirement. It is then possible, by the Theorem of Moments, to find the values of a_{11}, a_{12}, a_{21}, and a_{22}, for substitution into the equations

$$\begin{aligned} y_1 &= a_{11} P_1 + a_{12} P_2 \\ y_2 &= a_{21} P_1 + a_{22} P_2. \end{aligned} \tag{16}$$

If the shafts are cut at the inner support, and an equivalent bending moment is applied, a second form of these equations is obtained. Considering the individual shafts, in Fig. 5 (*b*) and (*c*), P_1 and P_2 are the equivalent concentrated loadings, and M is the bending moment at the intermediate support. Assuming, as above, that a_1 and a_2 are the unit deflections under the loads P_1 and P_2, and setting b_1 and b_2 equal to the deflections under the loads due to a unit bending moment, Equations 16 may be rewritten in the following manner:

$$\begin{aligned} y_1 &= a_1 P_1 + b_1 M, \\ y_2 &= a_2 P_1 - b_2 M. \end{aligned} \tag{17}$$

Then, from Fig. 5 (*b*) and (*c*), the values of the unit deflections are observed to be

$$\begin{aligned} a_1 &= \frac{d_1}{P_1}, \qquad a_2 = \frac{d_2}{P_2}, \\ b_1 &= \frac{e_1}{M}, \qquad b_2 = \frac{e_2}{M}. \end{aligned} \tag{18}$$

Solving for the value of the bending moment M, since the shaft is continuous,

$$\theta_{P_1} + \theta_{M_1} = \theta_{P_2} - \theta_{M_2}, \tag{19}$$

where from Fig. 5 (*b*),

$$\theta_{P_1} = \beta_1 P_1 \quad \text{and} \quad \theta_{M_1} = \mu_1 M,$$

and, from (*c*),

$$\theta_{P_2} = \beta_2 P_2 \quad \text{and} \quad \theta_{M_2} = \mu_2 M.$$

Substituting in Equation 19,

$$\beta_1 P_1 + \mu_1 M = \beta_2 P_2 - \mu_2 M, \tag{20}$$

from which

$$M = \frac{-\beta_1 P_1 + \beta_2 P_2}{\mu_1 + \mu_2}. \tag{21}$$

Further, substituting back into Equations 17, we obtain

$$\begin{aligned} y_1 &= \left[a_1 - \left(\frac{b_1 \beta_1}{\mu_1 + \mu_2} \right) \right] P_1 + \left(\frac{b_1 \beta_2}{\mu_1 + \mu_2} \right) P_2 \\ y_2 &= \left(\frac{b_2 \beta_1}{\mu_1 + \mu_2} \right) P_1 + \left[a_2 - \left(\frac{b_2 \beta_2}{\mu_1 + \mu_2} \right) \right] P_2, \end{aligned} \tag{22}$$

which are equivalent to Equations 16. The coefficients of P_1 and P_2, in Equations 22, determine the values of the influence factors for the shaft system, a_{11}, a_{12}, a_{21}, and a_{22}.

From Maxwell's Law for reciprocal displacements, it is readily seen that a_{12} should equal a_{21} for this type of mechanical system. In the above case, from Equations 22, it is required for equivalence that

$$b_1 = \beta_1 \quad \text{and} \quad b_2 = \beta_2.$$

This is, of course, true. A simple proof of this condition is found by considering shafts of uniform section with concentrated loads acting at the middle and moments applied at one end, where, from the ordinary beam equations, the influence factors are

$$b = \beta = \frac{l^2}{16EI}.$$

Continuing, if we let

$$P_1 = M_1 y_1 \omega^2 \quad \text{and} \quad P_2 = M_2 y_2 \omega^2, \tag{23}$$

where M = Mass = P/g, and if we substitute the values of the influence factors from Equations 22, and the Equations 23 into Equations 16, we obtain expressions similar to Equations 13 in the Introduction. The solution of these expressions determines the speeds at which critical vibration will occur.

b. General Solution for a Shaft with Many Supports. For a shaft having n shafts rigidly coupled together, as in Fig. 6, the displacement equations, from Fig. 6 (a), are extensions of Equations 16, as follows:[9]

$$\begin{aligned} y_1 &= a_{11}P_1 + a_{12}P_2 + \cdots a_{1k}P_k + \cdots a_{1n}P_n, \\ y_2 &= a_{21}P_1 + a_{22}P_2 + \cdots a_{2k}P_k + \cdots a_{2n}P_n, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_n &= a_{n1}P_1 + a_{n2}P_2 + \cdots a_{nk}P_k + \cdots a_{nn}P_n; \end{aligned} \tag{24}$$

and the influence factors, $a_{11}, \cdots a_{kk}, \cdots a_{nn}$, are similarly obtained by the statically indeterminate methods of the Theorem of Moments. The shafts are cut at the supports, and equivalent bending moments are applied. Considering

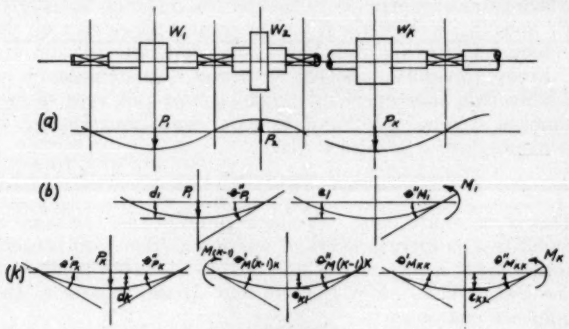


FIGURE 6

the individual shafts, as in Fig. 6 (b) and (k), let $a_1, \cdots a_k, \cdots a_n$, be the unit deflections under the loads, and $b_1, \cdots b_k, \cdots b_n$, be the unit deflections for the moments; then Equations 24 take the form[10]

$$\begin{aligned} y_1 &= a_1P_1 + b_1M_1, \\ &\cdots\cdots\cdots\cdots\cdots \\ y_k &= a_kP_k - b_{k1}M_{k-1} - b_{k2}M_k, \\ &\cdots\cdots\cdots\cdots\cdots \\ y_n &= a_nP_n + b_nM_{n-1}. \end{aligned} \tag{25}$$

[9] Lord Rayleigh, Scientific Papers, Vol. I, p. 197.

[10] *Applied Elasticity* (See footnote 3), Chap. V, pp. 100–104.

Again, from Fig. 6 (b) and (k), the values of the unit deflections are observed to be

$$a_1 = \frac{d_1}{P_1}, \qquad a_k = \frac{d_k}{P_k},$$
$$b_1 = \frac{e_1}{M_1}, \qquad b_{k1} = \frac{e_{k1}}{M_{k-1}}, \qquad b_{k2} = \frac{e_{k2}}{M_k}. \tag{26}$$

Solving for the bending moments $M_1, \cdots M_k, \cdots M_{n-1}$, since the shaft is continuous,

$$\theta''_{P_1} + \theta''_{M_1} = \theta'_{P_2} - \theta'_{M_{12}} - \theta'_{M_{22}},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\theta''_{P_{(k-1)}} + \theta''_{M_{(k-2)(k-1)}} + \theta''_{M_{(k-1)(k-1)}} = \theta'_{P_k} - \theta'_{M_{(k-1)k}} - \theta'_{M_{kk}}, \tag{27}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\theta''_{P_{(n-1)}} + \theta''_{M_{(n-2)(n-1)}} + \theta''_{M_{(n-1)(n-1)}} = \theta'_{P_n} - \theta'_{M_{(n-1)n}},$$

where, from Fig. 6 (b),

$$\theta''_{P_1} = \beta_1''P_1 \quad \text{and} \quad \theta''_{M_1} = \mu_{11}''M_1;$$

and, from (k)

$$\theta'_{P_k} = \beta_k'P_k \quad \text{and} \quad \theta''_{P_k} = \beta_k''P_k,$$

and

$$\theta'_{M_{(k-1)k}} = \mu'_{M_{(k-1)k}}M_{(k-1)}; \qquad \theta''_{M_{(k-1)k}} = \mu''_{M_{(k-1)_k}}M_{(k-1)};$$
$$\theta'_{M_{kk}} = \mu'_{M_{kk}}M_k; \qquad \theta''_{M_{kk}} = \mu''_{M_{kk}}M_k.$$

Substituting into Equations 27, the values of the angles in terms of the unit loads and moments, $n - 1$ equations are derived, and from the simultaneous solution of these equations, expressions for the moments, $M_1, \cdots M_k, \cdots M_{n-1}$, are obtained. The functional notation of these expressions may assume the form

$$M_1 = F_1(P_1, \cdots P_k, \cdots P_n),$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$M_k = F_k(P_1, \cdots P_k, \cdots P_n), \tag{28}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$M_{n-1} = F_{n-1}(P_1, \cdots P_k, \cdots P_n).$$

Substituting the values of the above moments into Equations 25, equations, equivalent to Equations 24, are derived, having real values for the influence factors a_{11}, a_{12}, $\cdots a_{kk}$, $\cdots a_{nn}$.

Then setting, (M = Mass)

$$P_1 = M_1 y_1 \omega^2; \qquad \cdots; \qquad P_n = M_n y_n \omega^2, \qquad (29)$$

and substituting the values of the influence factors and Equations 29 into Equations 24, we obtain n expressions for the n critical speeds of the shaft system.

2. *Consideration of Flexible Bearing Supports.*

Proceeding with the analysis of continuous shafts as statically indeterminate structures by the Theorem of Moments, we shall go to the more exact case of critical vibration where consideration is taken of the flexibility of bearing supports.[11] The spring constants of the pedestals are found by static deflection tests. In the machines to which this analysis has been applied, the masses of the bearing housings and the bearings are negligible compared to the other masses involved. It is thus assumed that the bearing supports act as connecting springs between the shaft structure and the foundation. The influence factors in this system, therefore, include the flexibility of the supports.

a. Application of Equations to a Shaft with Three Supports.—Since the bases of the pedestals are attached rigidly to the foundation, a vertical plane through the points of fixture will denote the center of equilibrium about which the shaft masses in vibration are constrained to move. For this case, a shaft system similar to that analyzed in the first part of the previous section, will be considered. From Fig. 7, the deflection curve of the continuous shaft for the fundamental mode of motion assumes the deformation shown in Fig. 7 (*a*). The general equations for this type of motion are the same as Equations 16.

Let P_1 and P_2 be the equivalent loads on the shafts, and let F_1, F_2, and F_3 be the reaction forces on the pedestals due

[11] Soderberg, C. R., "The Vibration Problem in Engineering," *The Electric Journal*, Vol. XXI, December 1924, p. 579.

to these loads. It is noted that, here, as before, the direction of action assumed for the loads, forces, and moments merely serve to make clear the dynamics of the problem, although in the analysis it is necessary that the assumptions are consistent throughout. Often, in actual problems, the signs in portions of the numerical equations are reversed, which gives evidence that some of the above factors have opposite senses.

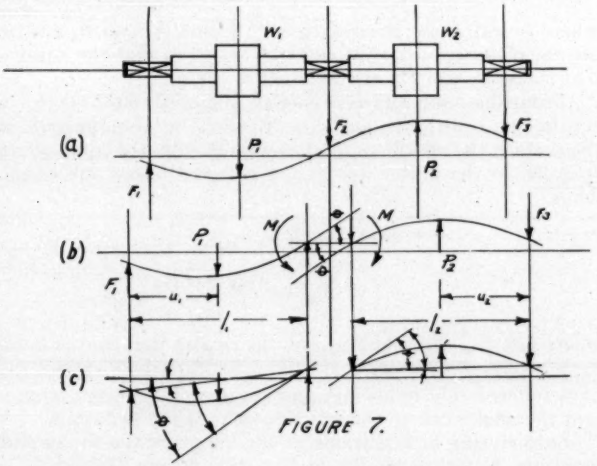


FIGURE 7.

From Fig. 7 (*b*) and (*c*), the deflections under the loads, P_1 and P_2, are

$$y_1 = P_1 a_1 + M b_1 + \frac{F_1 d_1 - F_2 d_2}{2},$$
$$y_2 = P_2 a_2 - M b_2 + \frac{F_2 d_2 + F_3 d_3}{2}, \qquad (30)$$

where, as in Fig. 5 (*b*) and (*c*) and Equations 17 and 18, a_1 and a_2 are the deflections under the loads due to unit weights, and b_1 and b_2 are the deflections under the loads due to unit bending moments; and as above, d_1, d_2, and d_3 are the deflections at the shaft centerline of the three pedestals due to unit reaction forces, respectively.

Considering the individual shafts, in Fig. 7 (*c*), the following equations for the moment at the middle support may be derived:

$$\begin{aligned} M &= F_1 l_1 - P_1 \frac{l_1}{u_1}, \\ &= F_3 l_2 - P_2 \frac{l_2}{u_2}, \end{aligned} \tag{31}$$

where l_1 and l_2 are the lengths of the shafts, and u_1 and u_2 are the distances from the external supports that the equivalent loads, P_1 and P_2, are assumed to act.

From the equivalence of angles, Fig. 7 (*b*) and (*c*), for a continuous shaft, the angles subtended by the tangent to the shaft at the middle support and the position of equilibrium (parallel to the center line of the pedestal bases) are equal. Thus,

$$\begin{aligned} \theta &= \theta_{P_1} + \theta'_M + \frac{F_1 d_1 + F_2 d_2}{l_1}, \\ &= \theta_{P_2} - \theta''_M + \frac{F_2 d_2 - F_3 d_3}{l_2}, \end{aligned} \tag{32}$$

where θ_{P_1}, θ_{P_2}, θ'_M and θ''_M are the angles due to the total loads and moments, respectively, as in Equations 19 and the data immediately following; and the remaining terms represent the angles due to the misalignment of the pedestals.

Substituting in Equations 32 the values of the forces and moments, equations similar to Equation 20 are derived:

$$\begin{aligned} \beta_1 P_1 + \mu_1 M + \frac{F_1 d_1 + F_2 d_2}{l_1} \\ = \beta_2 P_2 - \mu_2 M + \frac{F_2 d_2 - F_3 d_3}{l_2}. \end{aligned} \tag{33}$$

Since it is necessary to solve Equations 30 to determine the values of the influence factors in Equations 16, four equations for the four unknowns must be obtained. The equivalent of three equations are available in Equations 31 and 33; and the fourth relation may be established by a consideration of the external forces in equilibrium as follows:

$$F_1 + P_2 = P_1 + F_2 + F_3. \tag{34}$$

The solution of these equations may be obtained by substitution, for, from Equations 31, it is seen that

$$F_1 = \frac{P_1\left(\frac{l_1}{u_1}\right) - P_2\left(\frac{l_2}{u_2}\right) + F_3 l_2}{l_1}, \qquad (35)$$

and, from the combination of Equations 34 and 35,

$$F_2 = \frac{P_1\left(\frac{l_1}{u_1}\right) - P_2\left(\frac{l_2}{u_2}\right) + F_3 l_2}{l_1} - F_3 - P_1 + P_2. \qquad (36)$$

Then substituting Equations 31, 35, and 36 into Equations 33, the value of F_3 will be obtained in terms of P_1 and P_2, which may then be represented as the function of

$$F_3 = k_{31}P_1 + k_{32}P_2;$$

and

$$F_1 = k_{11}P_1 + k_{12}P_2, \qquad (37)$$

and

$$F_2 = k_{21}P_1 + k_{22}P_2,$$

will denote the values of F_1 and F_2 subsequently derived from substitution into Equations 35 and 36.

Substituting the value of F_1 into the first equation in 31 will give the value of the moment:

$$M = k_{M1}P_1 + k_{M2}P_2. \qquad (38)$$

It is only a step to complete the analysis, for all the terms in Equations 30 are now known, and the influence factors, a_{11}, a_{12}, a_{21}, and a_{22}, the coefficients of P_1 and P_2, are readily found. Proceeding as before from the similar equations, 8 and 9 in the Introduction, and solving in determinants, the roots of the equation of critical frequency, when reduced to revolutions per minute, are the critical speeds of the system.

b. General Solution for a Shaft with Many Supports.—It will be at once understood that the equations for the determination of the critical speeds of a continuous shaft with a large number of spans, and having flexible supports, would become too complicated for a continuation of this problem in a general way. With this difficulty in mind, the solution

of the critical speeds for a very common type of electrical machine, one having four bearing supports, and three rotor masses, will be completely given in Appendix 1. The first part is a theoretical analysis, similar to the preceding section, for the equations of the statically indeterminate structure; and the second part gives the numerical calculations, showing the application of the equations to an actual machine.

It will be conceded, however, that this analysis may be extended to shafts having five or more spans, and carrying as many individual loads. For the solution of the reaction forces and the moments, $2n$ equations are necessary, where n is the number of spans. These are obtained by the equivalence of forces, the values of the moments in terms of forces, the equations of potential and kinetic energy, and such other means as are commonly used in the calculation of statically indeterminate structures. Often, in actual problems, various sections are similar, which simplifies the analysis to a large extent. It is also possible that, with use, certain factors may be considered relatively unimportant, and may be eliminated; thereby greatly enhancing the practical value of the solutions.

B. Solution by the Reciprocal Theorem.

The second method of analysis, in the derivation of the influence factors of multiple shaft systems, obtains a solution directly by means of the Reciprocal Theorem and certain related mathematical principles. Derivations of this type were originally investigated by Lord Rayleigh, and Ritz,[12] and the particular application to the problem of critical speeds was first explained by Borowicz. The specific case of the Reciprocal Theorem as developed by Maxwell, previously referred to, is also intimately connected with this analysis.

The present application of the Reciprocal Theorem is based upon the premise that a continuous shaft may be considered as a single shaft, with the intermediate supports acting as additional loads. This assumption eliminates the complications of the moments in the statically indeterminate problem, and presents a solution for the influence factors no more difficult than for single shafts. Specific and general solutions of the problem are given for both rigid and flexible supports.

[12] Ritz, W., *Annalen d. Phys.*, Vol. 88, pp. 135 and 205.

1. *Consideration of Rigid Bearing Supports.*

For this part of the analysis, it is assumed that the bearing pedestals are inflexible, thus no motion occurs at the points of support.

a. Application of Equations to a Shaft with Two Supports.—In the general case of a single load on a shaft—equivalent, in mechanics, to a weight freely suspended on a spring—the potential energy of the system, referring to Figs. 1 and 2, may be expressed as

$$U = \frac{Py}{2} = \frac{P^2 a}{2}. \tag{39}$$

By the application of the Theorem of Castigliano, which states [13] that the partial derivative of the potential energy of deformation with respect to an external force equals the displacement corresponding to that force, from Equation 39, we have

$$\frac{\partial U}{\partial P} = y = Pa, \tag{40}$$

where P is the load on the shaft, y is the static deflection, and a is the deflection for a unit weight.

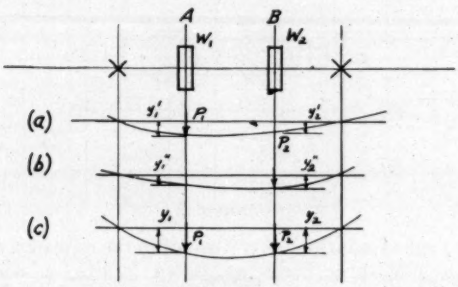


FIGURE 8.

Continuing the analysis, in Fig. 8 is shown the case of two loads on a shaft. Fig. 8 (*a*) is the deflection curve for the

[13] Castigliano, "Nuova Teoria Interno dell' Equilibrio dei Sistemi Elastici," Torino, *Atti della Academia delle scienze*, 1875, and "Theorie de l'equilibre des systemes elastiques," Turin, 1879.

load at point A, giving definite deflections at A and B of y_1' and y_2'. Fig. 8 (b) is the curve for the load at B, with the corresponding deflections of y_1'' and y_2''. The total deflection curve, Fig. 8 (c), is obtained by superimposing the two individual deflection curves. The potential energy, from Fig. 8 (c), is

$$U = \frac{P_1 y_1}{2} + \frac{P_2 y_2}{2} \tag{41}$$

from which the total deflections under the loads are, respectively,

$$\frac{\partial U}{\partial P_1} = y_1, \quad \text{and} \quad \frac{\partial U}{\partial P_1} = y_2,$$

where

$$\begin{aligned} y_1 &= y_1' + y_1'' = P_1 a_{11} + P_2 a_{12}, \\ y_2 &= y_2' + y_2'' = P_1 a_{21} + P_2 a_{22}, \end{aligned} \tag{42}$$

and a_{11}, a_{12}, a_{21}, and a_{22} are the influence factors of the system. The above equations are the expressions of the Reciprocal Theorem established by Maxwell, and the critical speeds for the shaft are determined in the same manner as in the analysis following Equations 8 and 9 of the Introduction.

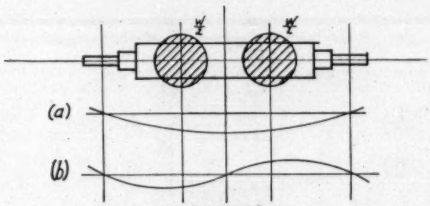


FIGURE 9.

This type of solution is very useful in the case of a machine having a long rotor body, as shown in Fig. 9, where the second critical speed—*i.e.*, the second mode of deformation, Fig. 9 (b)—may occur within the running range. For the purpose of analysis, the rotor weight is divided into two parts, and the equivalent weights of the two sections are considered to act at their respective centers of gravity. The rotor in this condition is identical with the shaft illustrated

in Fig. 8. The largest root of the second degree equation of the critical speeds, Equation 15, determines the second critical speed.

b. General Solution for a Shaft with Many Supports.—The general solution for the potential energy of a shaft with many supports involves the assumption that the reaction forces at the intermediate supports are equivalent to ordinary shaft loads. In Fig. 10 is shown a continuous shaft with $\frac{n+1}{2}$ spans, $\frac{n+1}{2}$ impressed loads, and $\frac{n-1}{2}$ reactions, where n is the total number of superimposed loads, neglecting the end reactions. The deflection curve, Fig. 10 (a), is

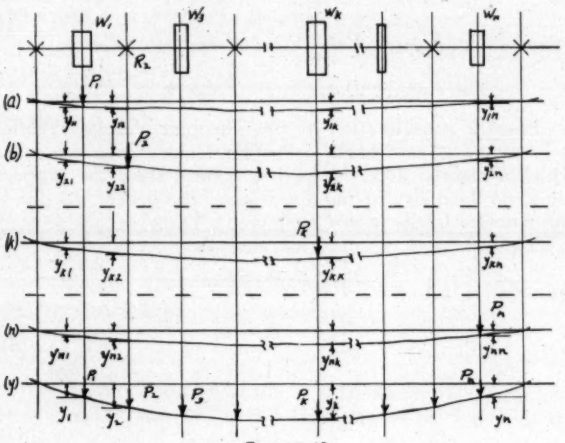


FIGURE 10.

obtained by considering the continuous shaft as a single shaft, and allowing the first shaft load to act alone. The deflections, y_{11}, y_{12}, $\cdots y_{1k}$, $\cdots y_{1n}$, are thereby incurred. This procedure may be continued for each shaft load, P_2, $\cdots P_k$, $\cdots P_n$, as illustrated in Fig. 10 (b), $\cdots$ (k), $\cdots$ (n). The summation of all the deflections, occurring at one time and in the same direction is represented in Fig. 10 (y), where

$y_1, y_2, \cdots y_k, \cdots y_n$ are the total deflections. Then, referring to Equation 41, the equation of the potential energy is

$$U = \frac{P_1 y_1}{2} + \frac{P_2 y_2}{2} + \cdots \frac{P_k y_k}{2} + \cdots \frac{P_n y_n}{2} \tag{43}$$

where the rotor loads have odd subscripts and the pedestal reactions have even subscripts.

Continuing as in the preceding section, the equations for the partial derivatives of the above equation, with respect to the individual loads, finally assume the form

$$\begin{aligned}
y_1 &= P_1 a_{11} + P_2 a_{12} + \cdots P_k a_{1k} + \cdots P_n a_{1n}, \\
y_2 &= P_1 a_{21} + P_2 a_{22} + \cdots P_k a_{2k} + \cdots P_n a_{2n}, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
y_k &= P_1 a_{k1} + P_2 a_{k2} + \cdots P_k a_{kk} + \cdots P_n a_{kn}, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
y_n &= P_1 a_{n1} + P_2 a_{n2} + \cdots P_k a_{nk} + \cdots P_n a_{nn}.
\end{aligned} \tag{44}$$

From a consideration of the Theorem of Least Work,[13] it is not necessary that a deflection occur at every point at which a reactive force is applied. Thus, since the supports allow no flexibility in the above case, the deflections due to the reactive loads, from Castigliano's Theorem, are

$$\begin{aligned}
\frac{\partial U}{\partial P_2} &= y_2 = 0, \\
\frac{\partial U}{\partial P_4} &= y_4 = 0, \\
&\cdots\cdots \\
\frac{\partial U}{\partial P_{(n-1)}} &= y_{(n-1)} = 0.
\end{aligned} \tag{45}$$

From Equations 45 it may be observed that $\frac{n-1}{2}$ of the equations in 44 are equal to zero, and it is possible, by a simultaneous solution of these equations, to obtain the values of the $\frac{n-1}{2}$ reactive forces in terms of the impressed loads. When these values are substituted into the remaining $\frac{n+1}{2}$

equations of the impressed loads, the following equations are derived:

$$\begin{aligned} y_1 &= P_1b_{11} + P_3b_{13} + \cdots P_kb_{1k} + \cdots P_nb_{1n}, \\ y_3 &= P_1b_{31} + P_3b_{33} + \cdots P_kb_{3k} + \cdots P_nb_{3n}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_n &= P_1b_{n1} + P_3b_{n3} + \cdots P_kb_{nk} + \cdots P_nb_{nn}, \end{aligned} \qquad (46)$$

where P_1, P_3, $\cdots P_n$ are the impressed loads on the shaft, and b_{11}, b_{13}, $\cdots b_{nn}$ are the influence factors. Equation 46 is a general form of Maxwell's Equation, and the solution for the critical speeds is performed in the same manner as has been previously given.

2. *Consideration of Flexible Bearing Supports*

For the second part of this analysis, it is assumed that the bearing pedestals are flexible; that a movement occurs at the support, proportional to the reaction, and in the opposite direction.

a. Application of Equations to a Shaft with Two Supports.—In the case of a single load on a shaft having flexible supports —equivalent, in mechanics, to a weight freely suspended on two springs in series—the equation for the static deflection is the same as Equation 40, although the value of y now includes an additional deflection due to the deformation of the pedestals. In Fig. 3, the displacement of the weight from the position of equilibrium is equivalent to

$$y = y' + y'' = Pa, \qquad (47)$$

where y' is the deflection of the weight relative to the centerline of the bearings, y'' is the deflection of the load due to the displacement of the supports, and a is the influence factor. Then,

$$y'' = \frac{R_1d_1l_2 + R_2d_2l_1}{l_1 + l_2} \qquad (48)$$

where R_1 and R_2 are the reactions at the supports, equivalent to

$$R_1 = \frac{Pl_2}{l_1 + l_2}, \quad \text{and} \quad R_2 = \frac{Pl_1}{l_1 + l_2},$$

d_1 and d_2 are the flexibility constants of the pedestals, and l_1 and l_2 denote the position of the load.

For the case of two loads on a shaft, Fig. 11, the diagrams (*a*) and (*b*) represent the first and second modes of deformation at the critical speeds, respectively. In Fig. 11 (*c*) and (*d*), as in Fig. 8 (*a*) and (*b*), y_1', y_2', y_1'', and y_2'', represent

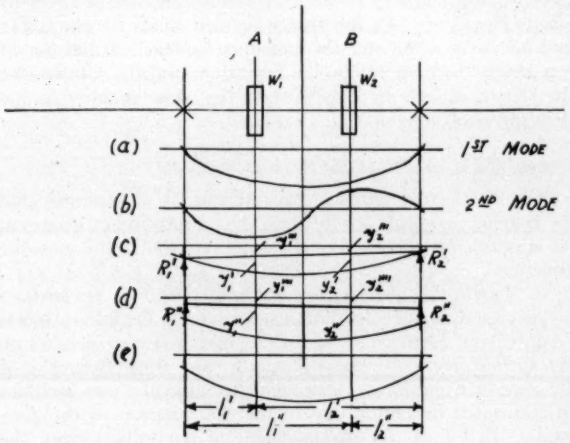


FIGURE 11.

deflections of the shaft alone; and in the former figure, the additional deflections, y_1''', y_2''', y_1'''', and y_2'''', are due to the movement of the supports. The latter deflections are equivalent to

$$y_1''' = \frac{R_1' d_1 l_2' + R_2' d_2 l_1'}{l},$$
$$y_2''' = \frac{R_1' d_1 l_2'' + R_2' d_2 l_1''}{l}, \text{ etc.,} \tag{49}$$

as in Equation 48.

The summation of these deflections, from Fig. 11 (*e*), follows:

$$y_1 = y_1' + y_1''' + y_1'' + y_1'''' = P_1 a_{11} + P_2 a_{12},$$
$$y_2 = y_2' + y_2''' + y_2'' + y_2'''' = P_1 a_{21} + P_2 a_{22}, \quad (50)$$

determining the equations from which the critical speeds are calculated.

b. General Solution for a Shaft with Many Supports.—The analysis for the influence factors of a shaft with many supports, considering the supports to be flexible, continues in the same manner as in the above solution for a single shaft with two loads. In Fig. 12 are detailed the deflection dia-

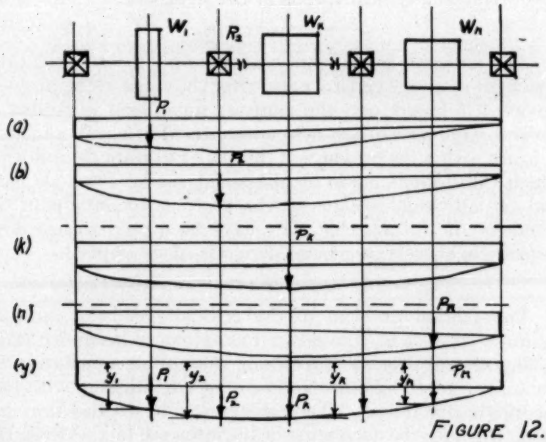


FIGURE 12.

grams for the individual conditions imposed upon the shaft, representing a combination of the factors involved in Figs. 10 and 11. The equations of potential energy, Equations 43, may be obtained from the summation curve shown in Fig. 12 (*y*). Equations 44 then represent the primary equations of deflection. It does not follow, however, that equations 45 are applicable; but, quite the contrary, a deflection does exist at each support, equivalent to the product of the reactive force and the flexibility constant of the pedestal. Thus, it may be established that

$$\begin{aligned} y_2 &= P_2 d_2, \\ y_4 &= P_4 d_4, \\ &\cdots \\ y_{(n-1)} &= P_{(n-1)} d_{(n-1)}, \end{aligned} \tag{51}$$

which are the equations for the pedestal reactions.

Proceeding as before, and replacing Equations 45 by Equations 51, it is again possible to obtain the final Equations 46. The solution of this equation, containing the influence factors and the shaft loads, by Maxwell's Reciprocal Theorem determines the critical speeds of the structure.

III. CONCLUSION.

The foregoing investigation admits only those conclusions which are of a very general nature, for the scope of the problem involved is broad, and the analyses have been expanded to cover a large part of the field encountered in actual practice. It is not within the province of this paper to state the influence which calculations made by these and similar methods have had on individual designs in the past, or to anticipate the trend for the future; that they have been used with a large measure of success in general design is alone implied.

A. General Conclusions from Theoretical Analyses.

The general problem of the critical speeds of multiple shafts is considered equivalent to that for a beam structure having several degrees of freedom, susceptible to solution by statically indeterminant methods in conjunction with the theories of dynamics. The problem may be divided into two parts; the first, the derivation of the influence factors from the flexibility constants of the individual units in the system, and the second, the solution of the equations containing the influence factors for the determination of the critical speeds. For the first part, graphical solutions are necessary, in general to obtain the shaft flexibilities; experimental data are required for the measurements of pedestal rigidity; and the equations of continuity are used to express the relationship of the various physical conditions that are effective in the solution for the influence factors. For the second part, the expressions for the Theorem of Maxwell consistently point the way to the termination of the problem.

The equations derived in the analyses may be applied directly in calculations of a character similar to those given. In many machines having multiple shafts, certain spans are identical, and this factor greatly simplifies the calculations.

B. Remarks on Example Calculations.

1. *Comparison of Machine Calculations.*

Tabulated summaries of the results of the machine calculations have been given for each of the four-bearing units taken as examples of the application to concrete problems of the theoretical analyses set forth. For the set having three units, the first critical speed is not lowered to a great extent by the introduction of flexible bearing supports, while for the second and third critical speeds, a large reduction is noted. For the set having two units, the critical speeds are lowered to approximately 60 per cent. of the former value. It is understood, however, that many factors of construction enter into these calculations, and that any other individual design may give widely different results. The numerical substitutions given in the solution of the example afford a valuable comparison for similar calculations.

2. *Calculation of Errors.*

The error in the determination of the critical speeds of multiple shafts is considerably greater than for single shafts. The graphical analysis of the deflection curve is correct to about 1 per cent. Assuming that a slide rule is used, as in the first method, with an accuracy of 0.5 per cent., the whole calculation may be considered correct to within 5 per cent. In the second method, using five place logarithms, while the calculations themselves have no error, the accumulation of the errors of the constants may give a final error of 10 per cent., although the second method is usually considered as accurate as the first.

3. *Methods in Practical Use.*

The first method given is perhaps the most applicable for use in design for several reasons, some of which are as follows: First, it allows an individual analysis of the conditions ob-

tained for each span—the calculation of the maximum deflection, the critical frequency, and the maximum stress. Second, the analysis entails a slide rule calculation with sufficient accuracy. Third, the effect of a change in design may readily be taken into account.

The second method has the advantage in that it affords a simple, straight-forward solution, the theoretical treatment of which is more easily grasped than the first. Its greatest use, however, may be found in systems more complicated than those treated in this paper.

C. Continuation of Investigation.

This paper may serve merely to point out the way for further investigation of this problem, which, although more empirical in nature, and perhaps more approximate, will simplify the calculations by such an extent as to make them applicable to all types of machines. Other methods of solution may be found to carry on the problem from the graphical analysis of the flexibility constants, and direct solutions of the simultaneous equations involved will greatly simplify the problem.

It is important that experimental data be obtained by the use of models and through actual field tests with which to compare the calculations. Especially in that phase of the problem concerned with flexible bearing supports is there very little data available. It is to be expected that in the future much work will be done in the investigation of this problem, for recognition of its importance is now made in design.

APPENDIX I.

An example of the solution for the critical speeds of a shaft with four supports by the Theorem of Moments is given as an illustration of the general solution of a shaft [14] with many supports. The specific equations and calculations are given for each of the two conditions of support;—the first, for the consideration that the bearing pedestals are rigid, and the second, for the consideration that the bearing supports are flexible. The analysis of the example machine, a large motor-

[14] *Steam and Gas Turbines* (See footnote 1), Art. 91, p. 440, and Art. 92, p. 449. *Applied Elasticity* (See footnote 3), Chap. IV, p. 82.

generator set having three dissimilar units, is based upon the actual design data from which the machine was built.

The analysis for the first condition, considering the supports to be rigid, follows:

The diagrammatic representation of this machine is given in Fig. 13. The assumed deflection curve for the fundamental critical speed, Fig. 13 (*a*), is taken from the general case shown

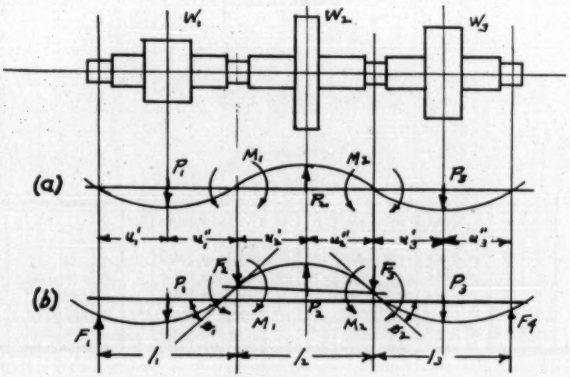


FIGURE 13

in Fig. 6 (*a*). The general Equations 25 now take the form, for the specific case above,—

$$\begin{aligned} y_1 &= a_1P_1 + b_1M_1 \\ y_2 &= a_2P_2 - b_{21}M_1 - b_{22}M_2, \\ y_3 &= a_3P_3 + b_3M_2, \end{aligned} \tag{52}$$

in agreement with the preceding part.

The equivalent angles for the continuous shaft under consideration, from Figs. 6 (*b*) and (*k*), and Equations 27, are found to be—

$$\begin{aligned} \theta''_{P_1} + \theta''_{M_1} &= \theta'_{M_2} - \theta'_{M_{12}} - \theta'_{M_{22}}, \\ \theta'_{P_3} + \theta'_{M_2} &= \theta''_{P_2} - \theta''_{M_{12}} - \theta''_{M_{22}}. \end{aligned} \tag{53}$$

Substituting the values of the angles in terms of forces, moments, and flexibility constants in Equations 53 (as described in the above references) Equations 54 are obtained,

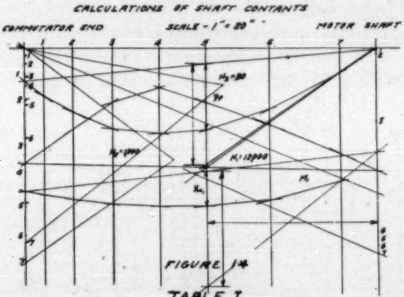


FIGURE 14

TABLE I

	SHAFT DATA							P, LOADING				MOMENT M,			
STA.	D	L	ΣL	W	ΣW	I	$\frac{L}{I}$	M	$ML\frac{L}{I}$	ΣWi	y	M	Wi	ΣWi	y
1	7	8	8.5	100	100	120	7.1	8.8	229	229		.005	3.02	3.02	
2	9	8	16.5	150	250	320	2.65	9.7	206	435		.14	2.97	5.99	
3	10.5	15	31.5	4800	5050	600	1.42	17.6	376	811		.25	5.34	11.33	
4	11	8	42.0	7730	12780	715	1.2	21.9	224	1035		.38	3.88	15.21	
5	11.5	16	66.0	500	13280	850	1.0	18.0	288	1323	17.9	.52	8.32	23.53	9.6
6	11.5	20	76.0	600	13880	850	1.0	12.0	240	1563		.70	14.00	37.53	
7	9	18	94.0	320	14200	320	2.65	4.4	210	1773		.90	43.00	80.53	

$$C_{P_1} = \frac{12000 \times 1200}{30 \times 10^6 \times 850 \times 14000} = 4.18 \times 10^{-6}(P_1)$$

$$C_{M_1} = \frac{80}{30 \times 10^6 \times 850} = 0.314 \times 10^{-6}(M_1)$$

$$\alpha_1 = 4.18 \times 10^{-6} \quad \times 17.9 = 75.0 \times 10^{-6}(P_1)$$

$$\beta_1 = 4.18 \times 10^{-6} \quad \times \frac{77.9}{45.0} = 2.6 \times 10^{-6}(P_1)$$

$$\gamma_1 = 0.314 \times 10^{-6} \quad \times 9.6 = 3.0 \times 10^{-6}(M_1)$$

$$\mu = 0.314 \times 10^{-6} \quad \times \frac{22}{45} = 0.222 \times 10^{-6}(M_1)$$

$$\begin{aligned} \beta_1''P_1 + \mu_{11}''M_1 &= \beta_2'P_2 - \mu_{12}'M_1 - \mu_{22}'M_2, \\ \beta_3'P_3 + \mu_{23}'M_2 &= \beta_2''P_2 - \mu_{12}''M_1 - \mu_{22}''M_2, \end{aligned} \tag{54}$$

from which equations the solutions for the moments are

$$M_1 = \frac{(-\beta_1''P_1+\beta_2'P_2)(\mu_{23}'+\mu_{22}'')-\mu_{22}'(\beta_2''P_2-\beta_3'P_3)}{(\mu_{11}''+\mu_{12}')(\mu_{23}'+\mu_{22}'')-\mu_{22}'\mu_{12}''},$$

$$M_2 = \frac{(\mu_{11}''+\mu_{12}')(\beta_2''P_2-\beta_3'P_3)-\mu_{12}''(-\beta_1''P_1+\beta_2'P_2)}{(\mu_{11}''+\mu_{12}')(\mu_{23}'+\mu_{22}'')-\mu_{22}'\mu_{12}''}. \qquad (55)$$

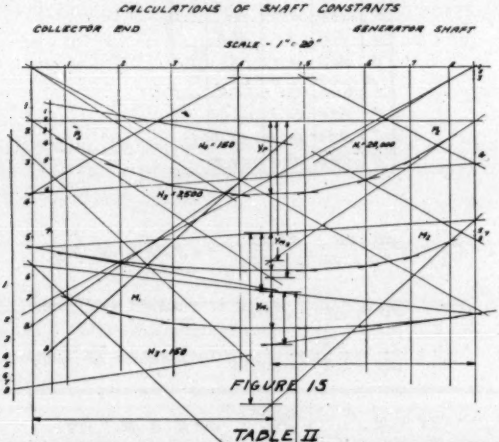


FIGURE 15

TABLE II

SHAFT DATA								P_1 LOADING				MOMENT M_1			
STA	D	L	ΣL	W	ΣW	I	$\frac{L}{I}$	M	$ML\frac{L}{I}$	ΣW_1	y	M	W_1	ΣW_1	y
1	9	15.50	15.5	240	240	322	2.88	69	533	533		.925	100.5	100.5	
2	12	12.62	28.12	450	690	1018	2.21	123	340	877		.807	220	320.5	
3	13.5	16.25	44.37	600	1290	1620	1.39	196	443	1320		.685	156	476.6	
4	16.5	18.25	62.62	1080	2370	2170	1.04	281	534	1854	281	.537	102	578.6	19
5	16.62	17.25	79.87	10830	22980	2250	1.00	271	468	2322		.388	66	1644	
6	12.0	14.0	93.87	480	23460	1018	2.21	174	551	2873		.245	76	1720	
7	10.75	9.0	102.87	230	23690	640	3.41	105	323	3196		.146	45	1675	
8	9.0	12.38	115.25	210	23900	322	7.00	40	346	3542		.052	45	1720	

TABLE II

The graphical solutions of the deflection curves, by the method developed by Morley, for one end shaft and the middle shaft, are given in Figs. 14 and 15, and Tables I and II.[15] The appended calculations determine the individual

[15] Morley, A., *Engineering*, Vol. 88, pp. 135 and 205.

CALCULATION OF SHAFT CONSTANTS

TABLE II

STA.	MOMENT M_2			
	M	W_i	ΣW_i	y
1	.053	5.8	5.8	
2	.192	5.4	11.2	
3	.315	7.1	18.3	
4	.466	8.8	27.1	10.2
5	.618	10.7	37.8	
6	.752	23.3	61.1	
7	.855	26.3	87.4	
8	.945	81.9	169.3	

$$C_{P_2} = \frac{20.000 \times 2500}{30 \times 10^6 \times 2250} = 3.22 \times 10^{-8}(P_2)$$

$$C_{M_1} = \frac{150}{30 \times 10^6 \times 2250} = 0.222 \times 10^{-8}(M_1)$$

$$C_{M_2} = \frac{150}{30 \times 10^6 \times 2250} = 0.222 \times 10^{-8}(M_1)$$

$$\alpha_2 = 3.22 \times 10^{-8} \times 20.1 = 64.8 \times 10^{-8}(P_2)$$

$$\beta_{21} = 3.22 \times 10^{-8} \times \frac{43.5}{62.1} = 2.25 \times 10^{-8}(P_2)$$

$$\beta_{22} = 3.22 \times 10^{-8} \times \frac{36.7}{53.2} = 2.34 \times 10^{-8}(P_2)$$

$$\nu_{21} = 0.222 \times 10^{-8} \times 10 = 2.22 \times 10^{-8}(M_1)$$

$$\nu_{22} = 0.222 \times 10^{-8} \times 10.2 = 2.26 \times 10^{-8}(M_2)$$

$$\mu_{11} = 0.222 \times 10^{-8} \times \frac{55.1}{62.1} = 0.197 \times 10^{-8}(M_1)$$

$$\mu_{12} = 0.222 \times 10^{-8} \times \frac{16.3}{53.2} = 0.060 \times 10^{-8}(M_1)$$

$$\mu_{21} = 0.222 \times 10^{-8} \times \frac{15.6}{62.1} = 0.056 \times 10^{-8}(M_2)$$

$$\mu_{22} = 0.222 \times 10^{-8} \times \frac{47.3}{53.2} = 0.197 \times 10^{-8}(M_2)$$

flexibility constants for the shafts, which are listed in Table III, following.

TABLE III.

Section.	Load Characteristics.		Moment Characteristics.	
1	$a_1 = 75.0$	$\beta_1'' = 2.60$	$b_1 = 3.00$	$\mu_{11}'' = 0.223$
2	$a_2 = 64.8$	$\beta_2' = 2.25$	$b_{21} = 2.22$	$\mu_{12}' = 0.197$
		$\beta_2'' = 2.34$	$b_{22} = 2.26$	$\mu_{22}' = 0.060$
				$\mu_{12}'' = 0.056$
				$\mu_{22}'' = 0.147$
3	$a_3 = 89.0$	$\beta_3' = 3.30$	$b_3 = 3.80$	$\mu_{23}' = 0.250$

Multiplier 10^{-8}

Supplying the real values of the flexibility constants from Table III, the equations of the moments in terms of the forces, Equations 55, become

$$\begin{aligned} M_1 &= -6.26\,P_1 + 4.73\,P_2 + 0.997\,P_3, \\ M_2 &= 0.844\,P_1 + 4.50\,P_2 - 7.50\,P_3. \end{aligned} \tag{56}$$

Substituting back into Equations 52, and employing the remaining flexibility constants, the equations of the deflections for the three shafts, combined, at the points of greatest load are

$$\begin{aligned} (10^8)y_1 &= 56.2\,P_1 + 14.12\,P_2 + 2.99\,P_3, \\ (10^8)y_2 &= 12.0\,P_1 + 44.1\,P_2 + 14.8\,P_3, \\ (10^8)y_3 &= 3.21\,P_1 + 17.1\,P_2 + 60.5\,P_3, \end{aligned} \tag{57}$$

where the coefficients of the loads are the influence factors.

The error observed in the disagreement of a_{12} and a_{21}, and a_{13} and a_{31}, and a_{23} and a_{32} originates in the graphical solution of the flexibility constants, but as Maxwell's Law may serve here to indicate the accuracy of the calculation, the graphical values will remain unchanged.

Setting

$$\begin{aligned} P_1 &= M_1 y_1 \omega^2, \\ P_2 &= M_2 y_2 \omega^2, \\ P_3 &= M_3 y_3 \omega^2, \end{aligned}$$

where the value of the individual loads, in terms of mass,

(W/g) are $M_1 = 35$, $M_2 = 60$, and $M_3 = 32$, Equations 57 become

$$\begin{aligned} 10^8 y_1 &= 1970\,\omega^2 y_1 + 846\,\omega^2 y_2 + 96\,\omega^2 y_3, \\ 10^8 y_2 &= 420\,\omega^2 y_1 + 2640\,\omega^2 y_2 + 474\,\omega^2 y_3, \\ 10^8 y_3 &= 112\,\omega^2 y_1 + 1027\,\omega^2 y_2 + 1940\,\omega^2 y_3. \end{aligned} \tag{58}$$

Solving by determinants, and setting the determinant in the denominator equal to zero,

$$0 = \begin{vmatrix} 1970\,\omega^2 - 10^8 & 846\,\omega^2 & 96\,\omega^2 \\ 420\,\omega^2 & 2640\,\omega^2 - 10^8 & 474\,\omega^2 \\ 112\,\omega^2 & 1027\,\omega^2 & 1940\,\omega^2 - 10^8 \end{vmatrix} \tag{59}$$

the equation of the critical frequencies becomes

$$8518\,\omega^6 - 1330\,\omega^4 + 65.5\,\omega^2 - 1 = 0. \tag{60}$$

Simplifying, and solving, the three roots of the equation are

$$\begin{aligned} \omega_1 &= 173, \\ \omega_2 &= 234, \\ \omega_3 &= 268, \end{aligned}$$

in radians per second, which are equivalent to critical speeds of 1650, 2240, and 2560, in revolutions per minute, respectively.

The analysis for the second condition, considering the supports to be flexible, for the above machine, follows:

Referring to Fig. 7, the specific case of a continuous shaft with three supports, Fig. 13 (*b*) represents the general case by the consideration of a shaft with four supports. It is necessary to develop equations similar to Equations 30 and those immediately following for use in the solution of the present example. From Fig. 13 (*b*), and the combination of Equations 30 and 52, the general equations for the deflections are:

$$\begin{aligned} y_1 &= P_1 a_1 + M_1 b_1 + \frac{F_1 d_1 - F_2 d_2}{2}, \\ y_2 &= P_2 a_2 - M_1 b_{12} - M_2 b_{21} + \frac{F_2 d_2 + F_3 d_3}{2}, \\ y_3 &= P_3 a_3 + M_2 b_3 + \frac{F_4 d_4 - F_3 d_3}{2}. \end{aligned} \tag{61}$$

Considering the individual shafts, the equations for the moments at the two interior supports are

$$\begin{aligned} M_1 &= F_1 l_1 - P_1 \frac{l_1}{u_1''}, \\ M_2 &= F_4 l_3 - P_3 \frac{l_3}{u_3'} \end{aligned} \tag{62}$$

corresponding to Equations 31.

The equations for the angular displacements, from Equations 32 and 53, and Fig. 13 (*b*) are found to be

$$\begin{aligned} \theta_1 &= \theta''_{P_1} + \theta''_{M_1} + \frac{F_1 d_1 + F_2 d_2}{l_1} \\ &= \theta'_{P_2} - \theta'_{M_{12}} - \theta'_{M_{22}} - \frac{F_2 d_2 - F_3 d_3}{l_2}, \end{aligned} \tag{63}$$

$$\begin{aligned} \theta_2 &= \theta'_{P_3} + \theta'_{M_2} + \frac{F_4 d_4 + F_3 d_3}{l_3}, \\ &= \theta''_{P_2} - \theta''_{M_{12}} - \theta''_{M_{22}} + \frac{F_2 d_2 - F_3 d_3}{l_2}, \end{aligned}$$

which, when converted into terms of forces, angles, and flexibility constants, become

$$\begin{aligned} &\beta_1'' P_1 + \mu_{11}'' M_1 + \frac{F_1 d_1 + F_2 d_2}{l_1} \\ &\quad = \beta_2' P_2 - \mu_{12}' M_1 - \mu_{22}' M_2 - \frac{F_2 d_2 - F_3 d_3}{l_2}, \end{aligned} \tag{64}$$

$$\begin{aligned} &\beta_3' P_3 + \mu_{23}' M_2 + \frac{F_4 d_4 + F_3 d_3}{l_3} \\ &\quad = \mu_2'' F_2 - \mu_{12}'' M_1 - \mu_{22}'' M_2 + \frac{F_2 d_2 - F_3 d_3}{l_2}. \end{aligned}$$

For the solution of this problem, six equations, $(2n)$, are required. Four of these are given in Equations 62 and 64, and two more are obtained by considering the forces in equilibrium,

$$P_1 + P_3 + F_2 + F_3 = P_2 + F_1 + F_4, \tag{65}$$

and by cutting the shaft at the position of the middle load, and equating the moments on each side—

$$F_1(l_1 + u_2') - P_1(u_1'' + u_2') - F_2u_2' = F_4(l_3 + u_2'') - P_3(u_3' + u_2'') - F_3u_2''. \quad (66)$$

The simultaneous solution of these equations is performed by substituting Equations 62 into Equations 64, and solving by determinants with Equations 65 and 66. Four resulting equations are obtained similar to Equations 37 in terms of the reactive forces and the shaft loads. When the values of F_1 and F_4 are substituted into Equations 62, all the factors in Equation 61 are known with respect to P_1, P_2, and P_3. The equations are now in the form required for the solution of the critical speeds.

The values of the flexibility constants, found from the graphical solutions of the individual shafts in the preceding section, are given in Table III. In addition to these factors, it is necessary to consider the flexibility constants of the pedestals. These constants are found by actual deflection measurements. In Fig. 16 is shown a sheet from the data book of the pedestal deflection test of the outboard pedestal No. 1. The values for the lateral deflections of the pedestals of the machine under discussion for unit loads applied at the shaft are given in Table IV.

TABLE IV.

Pedestal.	Deflection "/#
1	d_1 = 100
2	d_2 = 60
3	d_3 = 50
4	d = 90

Multiplier 10^{-8}

It is assumed that the deflection of the shaft is similar to that denoted in Fig. 13 (*b*), although for any other deflection curve likely to be obtained, the equations are equally correct.

The calculation is continued by substituting the numerical values of the flexibility constants from Tables III and IV, and the data from Figs. 13, 14, and 15, into the equations above.

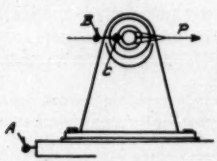
PEDESTAL DEFLECTION TEST - OUTBOARD PEDESTAL #1

4 BEARING - 3 UNIT M.G. SET - LATERAL MOTION

TEST DATA

* UNIT DIVISIONS = 92#

DIAL	MILLI-GAGE		STRAIN GAGE *				CALCULATIONS			
A	B	C	1	2	3	4	TOTAL	DIFF	LOAD	DEFL.
0	0	0	179	177	154	156	666	0	0	0
0	0.1	0	180	176	155	156				
0	1.0	1.0	182	164	166	184				
0	1.0	1.0	182	165	165	184	696	30	690	.001
0.1	2.15	2.5	186	158	184	218				
	2.1	2.4	186	158	186	217	774	81	1860	.0021
0.3	3.0	4.3	189	151	202	244				
0.4	2.95	4.2	189	151	202	244	786	120	2760	.003
0.0	1.5	2.5	182	162	188	203				
0.1	1.6	2.6	182	162	188	204	737	71	1630	.0016
0.0	0.25	0.5	177	176	153	153				
			177	177	154	154	662	-4	-92	.00025



A - GAGE ON BED PLATE
B - GAGE ON PEDESTAL AT ℄ BEARING
C - GAGE ON SHAFT

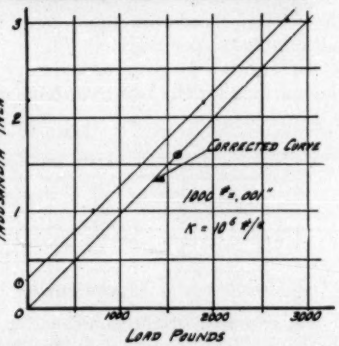


FIGURE 16

The reaction forces in terms of the span loads are found to be:

$$\begin{aligned} F_1 &= +\ 0.568\ P_1 - 0.0502\ P_2 - 0.0106\ P_3, \\ F_2 &= -\ 0.508\ P_1 + 0.410\ P_2 + 0.064\ P_3, \\ F_3 &= +\ 0.066\ P_1 + 0.490\ P_2 - 0.488\ P_3, \\ F_4 &= -\ 0.0095\ P_1 - 0.0514\ P_2 + 0.586\ P_3. \end{aligned} \tag{67}$$

Solving Equations 62, the equations for the moments are

$$\begin{aligned} M_1 &= +\ 6.300\ P_1 - 4.730\ P_2 - 1.00\ P_3, \\ M_1 &= -\ 0.835\ P_1 - 4.530\ P_2 + 7.50\ P_3. \end{aligned} \tag{68}$$

It is now necessary to substitute Equations 67 and 68, and the factors from Table III, into Equations 61 to obtain a solution similar to Equation 57 and the data following. The expression for the critical frequency is found to be

$$42024 \times 10^6\omega^6 - 3651 \times 10^{12}\omega^4 + 105.1 \times 10^{18}\omega^2 - 10^{24} = 0. \tag{68}$$

Solving for the three values of the critical frequencies of vibration, in radians per second,

$$\begin{aligned} \omega_1 &= 156, \\ \omega_2 &= 173, \\ \omega_3 &= 180.5, \end{aligned}$$

the critical speeds are 1490, 1650, and 1725, in revolutions per minute, respectively.

In Table V is given a summary of the critical speeds of the machine for the two conditions of structural support.

TABLE V.

Rigid Pedestals.	Flexible Pedestals.	Difference.	Per cent. Decrease.
1650	1490	160	9.7
2240	1650	590	26.3
2560	1725	835	32.6

APPENDIX II.

An example of the solution for the critical speeds of a shaft with four supports, by the methods of the Reciprocal Theorem, will be explained in the same manner as has been

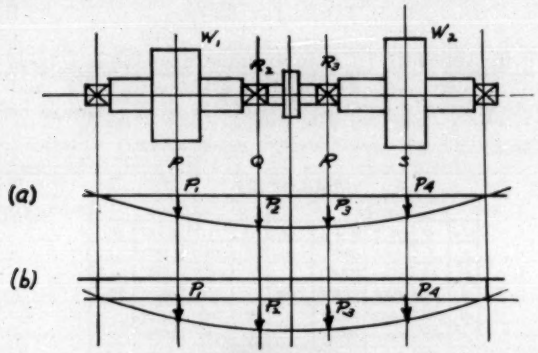


FIGURE 17

CALCULATION OF SHAFT CONSTANTS

SCALE - 1" = 60".

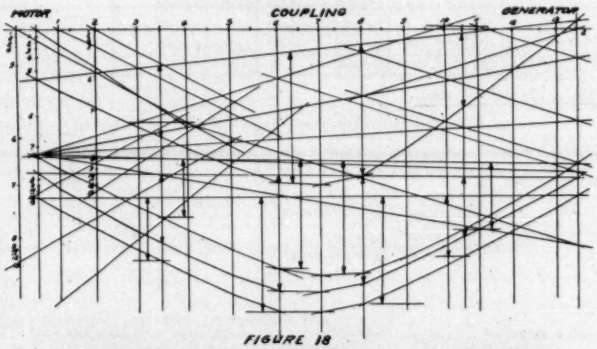


FIGURE 18

SHAFT DATA							P LOADING				Q LOADING			
STA	D	L	ΣL	W	I	$\frac{W}{I}$	M	$ML\frac{W}{I}$	ΣW₁	y₀	M	$ML\frac{W}{I}$	ΣW₁	y₀
1	16.5	31	31		3680	7.7	10	2390	2390		72	1730	1730	
2	22.0	32	63		11500	2.43	25	1950	4340		48.0	1400	3130	
3	27.5	42	105	100 000	28000	1.0	44	1850	6190	61.5	92.6	1358	4488	58.0
4	27.5	42	147	100 000	28000	1.0	82	2190	8370		44.0	2416	6900	
5	22	36.5	183.5		11500	2.43	46	3980	12350		73.2	6300	12200	
6	16	40.25	223.75	200000	3200	8.7	40	14000	26350	146.5	75.6	26400	38600	133.5
7	16	30.5	254.25		3200	8.7	38	9200	35570		64.8	17200	55800	
8	16	40.25	294.50	200000	3200	8.7	38	9300	45170	96.1	54.0	13000	68800	97.6
9	22	36.5	331.0		11500	2.43	29	2040	47210		43.2	3040	71840	
10	27.5	31.5	362.5	100000	28000	1.0	17	530	47740	53.6	33.6	1065	73025	58.0
11	27.0	31.5	394.0	100000	26100	1.07	19	660	48180		25.5	850	73135	
12	22.0	33.0	427.0		11500	2.43	9	720	48900		16.8	1240	74375	
13	16.5	31.0	458.		3680	7.7	9	720	49620		4.8	1150	73525	

TABLE VI

given in Appendix I. The example machine is a large frequency-changer set having two dissimilar units, and the analytical details are derived from the design data as before.

CALCULATION OF SHAFT CONSTANTS

TABLE VI

R LOADING					S LOADING			
STA	M	$ML\frac{\Delta\theta}{g}$	ΣW_i	y_R	M	$ML\frac{\Delta\theta}{g}$	ΣW_i	y_S
1	5	1320	1320		3	790	790	
2	13	1010	2330		7	540	1330	
3	23	960	3290	63.9	12	500	1830	53.6
4	34	1430	4720		18	760	2590	
5	44	3910	8630		24	2130	4720	
6	55	19300	27930	117.2	29	10200	14920	98.1
7	64	17000	44930		34	9050	23970	
8	74	26000	70930	119.4	40	14000	37970	101.3
9	59	5240	76170		45	4000	41970	
10	45	1420	77590		50	1580	43550	
11	34	1150	78740	67.1	45	1520	45070	60.1
12	22	1180	79920		29	1560	46630	
13	8	1910	81830		10	2390	49020	

$$C_p = \frac{3\times10^{-3}\times40\times10^{3}}{3\times10^{7}\times28\times10^{3}\times2\times10^{6}} = 7.14\times10^{-8}$$

$$C_q = \frac{3\times10^{-3}\times72\times10^{3}}{3\times10^{7}\times28\times10^{3}\times2\times10^{6}} = 12.85\times10^{-8}$$

$$C_r = \frac{3\times10^{-3}\times60\times10^{3}}{3\times10^{7}\times28\times10^{3}\times2\times10^{6}} = 10.72\times10^{-8}$$

$$C_s = \frac{3\times10^{-3}\times40\times10^{3}}{3\times10^{7}\times28\times10^{3}\times2\times10^{6}} = 7.14\times10^{-8}$$

$\alpha_{11} = 439\times10^{-8}$ $\alpha_{12} = 746\times10^{-8}$ $\alpha_{13} = 686\times10^{-8}$ $\alpha_{14} = 383\times10^{-8}$

$\alpha_{21} = 746\times10^{-8}$ $\alpha_{22} = 1331\times10^{-8}$ $\alpha_{23} = 1235\times10^{-8}$ $\alpha_{24} = 697\times10^{-8}$

$\alpha_{31} = 686\times10^{-8}$ $\alpha_{32} = 1255\times10^{-8}$ $\alpha_{33} = 1279\times10^{-8}$ $\alpha_{34} = 721\times10^{-8}$

$\alpha_{41} = 383\times10^{-8}$ $\alpha_{42} = 697\times10^{-8}$ $\alpha_{43} = 721\times10^{-8}$ $\alpha_{44} = 429\times10^{-8}$

The analysis for the critical speeds, considering the bearing supports to be rigid in the first case, is made in the following manner: [16]

[16] Soderberg, C. R., "Practical Application of the Theory of Vibrations to Systems with Several Degrees of Freedom," *Phil. Mag.*, Vol. V, January 1928.

This machine is represented diagrammatically in Fig. 17, which is a specific case of the general structure shown in Fig. 10. The summation of the deflection curves is represented in Fig. 17 (*a*) as in Fig. 10 (*y*). It is necessary to determine the flexibility constants in Equations 44 by the graphical constructions given in Fig. 18, and the calculations from Table VI. The values of the flexibility constants are listed in Table VII.

TABLE VII.

Position.	Loading.			
	P.	Q.	R.	S.
P	$a_{11} = 439$	$a_{12} = 746$	$a_{13} = 686$	$a_{14} = 383$
Q	$a_{21} = 746$	$a_{22} = 1331$	$a_{23} = 1255$	$a_{24} = 697$
R	$a_{31} = 686$	$a_{32} = 1255$	$a_{33} = 1279$	$a_{34} = 721$
S	$a_{41} = 383$	$a_{42} = 697$	$a_{43} = 721$	$a_{44} = 429$

Multiplier 10^{-8}.

Since there is no load on the shaft between the two interior pedestals, the pedestal loads, Q and R, are considered to act at positions P_2 and P_3, in place of the positions having odd subscripts as given in the analysis for the general case.

Setting up Equations 44 as follows:

$$\begin{aligned} 10^8\, y_1 &= 439\, P_1 + 746\, P_2 + 686\, P_3 + 383\, P_4, \\ 10^8\, y_2 &= 746\, P_1 + 1331\, P_2 + 1255\, P_3 + 697\, P_4, \\ 10^8\, y_3 &= 686\, P_1 + 1255\, P_2 + 1279\, P_3 + 721\, P_4, \\ 10^8\, y_4 &= 383\, P_1 + 697\, P_2 + 721\, P_3 + 429\, P_4; \end{aligned} \tag{69}$$

and considering the pedestals to be rigid,

$$y_2 = 0 \quad \text{and} \quad y_3 = 0,$$

the general equation for the value of P_2 is obtained—

$$P_2 = -\frac{(a_{12}a_{33} - a_{13}a_{32})P_1 + (a_{42}a_{33} - a_{43}a_{32})P_4}{a_{22}a_{33} - a_{32}a_{23}}, \tag{70}$$

with a similar equation for the value of P_3.

Substituting the values of the flexibility constants from

Table VII into Equation 70,

$$P_2 = \frac{(746 \times 1279 - 686 \times 1255) P_1 + (697 \times 1279 - 721 \times 1255) P_4}{(1331 \times 1279 - 1255 \times 1255)}, \tag{71}$$

it is found that a very accurate calculation is required to reduce the errors involved in the processes indicated for the quantities in the brackets. Using 5-place logarithms, the solution for the above equation is

$$P_2 = -0.7317\, P_1 + 0.1052\, P_4,$$

and likewise

$$P_3 = 0.1818\, P_1 - 0.6667\, P_4. \tag{72}$$

Substituting Equations 72 into the first and last equations in Equations 69, the expressions for the Reciprocal Theorem of Maxwell, containing the influence factors, are obtained:

$$\begin{aligned} 10^8\, y_1 &= 17.85\, P_1 + 4.10\, P_4, \\ 10^8\, y_4 &= 4.07\, P_1 + 21.60\, P_4. \end{aligned} \tag{73}$$

Setting

$$\begin{aligned} P_1 &= M_1 y_1 \omega^2 = 375.8\, y_1 \omega^2, \\ P_4 &= M_4 y_4 \omega^2 = 388.7\, y_4 \omega^2, \end{aligned} \tag{74}$$

and solving Equation 73 with the determinant in the denominator equal to zero, the critical speeds of the system, in revolutions per minute, are found to be

$$\begin{aligned} N_1 &= 1020, \\ N_2 &= 1250. \end{aligned}$$

The analysis for the second case, considering the bearing supports to be flexible, is made as follows:

Referring to Fig. 12 (*y*), the total deflection curve for the general case of a shaft with many flexible supports, the deflection curve, Fig. 17 (*b*), may be obtained from the conditions of the specific case above. To derive the values of the flexibility constants for substitution into Equations 44, it is necessary to add the individual effects of the deflections of the exterior pedestals to each of the constants listed in Table VII. The values for the lateral deflections of the pedestals are given in Table VIII.

TABLE VIII.

Pedestal.	Deflection "#
1	$d_1 = 65$
2	$d_2 = 50$
3	$d_3 = 50$
4	$d_4 = 65$

Multiplier 10^{-8}.

From the shaft data given in Fig. 18, and Tables VI, VII, and VIII, the flexibility constants for this condition of the shaft structure are determinable, and are here listed in Table IX. Equations similar to 69 are set up with these values, and the second and third equations are eliminated when

$$y_2 = P_2 d_2 \quad \text{and} \quad y_3 = P_3 d_3,$$

as given in Equations 51.

TABLE IX.

Position.	Loading.			
	P.	Q.	R.	S.
P	$a_{11} = 481$	$a_{12} = 780$	$a_{13} = 715$	$a_{14} = 405$
Q	$a_{21} = 780$	$a_{22} = 1414$	$a_{23} = 1287$	$a_{24} = 728$
R	$a_{31} = 715$	$a_{32} = 1287$	$a_{33} = 1363$	$a_{34} = 757$
S	$a_{41} = 405$	$a_{42} = 728$	$a_{43} = 757$	$a_{44} = 473$

Multiplier 10^{-8}.

Solving these equations for the values of P_2 and P_3 in terms of P_1 and P_4 from the general equations 70,

$$\begin{aligned} P_2 &= -0.5274\, P_1 - 0.06645\, P_4, \\ P_3 &= -0.0262\, P_1 - 0.49270\, P_4, \end{aligned} \tag{75}$$

and substituting further,

$$\begin{aligned} 10^8\, y_1 &= 50.9\, P_1 + 1.0\, P_4, \\ 10^8\, y_4 &= 1.2\, P_1 + 51.6\, P_4. \end{aligned} \tag{76}$$

Substituting Equations 74 in Equations 76, the values of the critical speeds are found to be, in revolutions per minute,

$$\begin{aligned} N_1 &= 675, \\ N_2 &= 717. \end{aligned}$$

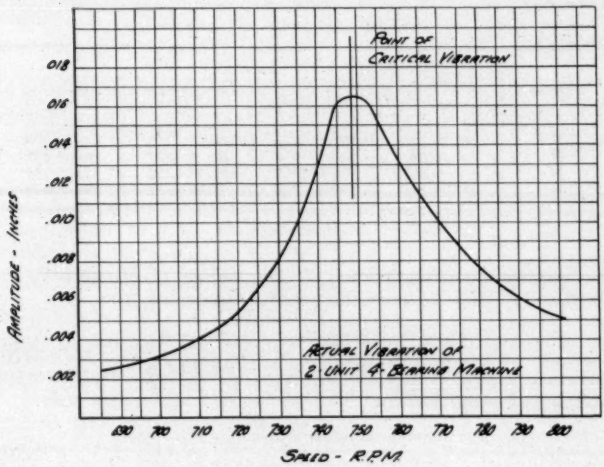
In Table X is given a summary of the critical speeds of the machine for the two conditions of structural support.

TABLE X.

Rigid Pedestals.	Flexible Pedestals.	Difference.	Per cent. Decrease.
1020	675	345	33.8
1250	717	533	42.6

As an indication of the influence of pedestal flexibility, in Fig. 19 is shown a sheet from a data book of vibration records taken on a machine with similar characteristics to the four-

FIG. 19.



bearing two-unit set in the above example. This machine was calculated, with rigid pedestals, to have a fundamental critical speed of about 1100 R.P.M., and the actual critical speed on the test floor was found to be reduced to 750 R.P.M.

ACKNOWLEDGMENT.

The author acknowledges indebtedness to Professor S. Timoshenko, Professor of Mechanics at the University of Michigan, and Mr. C. Richard Soderberg, Mechanical Engineer, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pennsylvania, for their kind suggestions and hearty coöperation in the development of this problem.

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The Goddard Rocket. Prof. Robert H. Goddard of Clark University, Worcester, Mass., has recently announced that his experiments in efficient rocket propulsion to reach extreme altitudes will be carried on in Roswell, New Mexico. Roswell has been chosen because of the climate, the nature of the country and the clearness of the air. The preliminaries attending the experiments will require considerable time and there is yet no definite idea as to just when actual work will begin.

The experiments have been carried on since 1912 with aid from the Smithsonian Institution, Clark University and Carnegie Institution. Last month it was announced that Mr. Daniel Guggenheim had made a grant for the continuation of the work, since the experiments give so much promise of valuable contributions to science in general.

The Goddard rocket is propelled by the thrust produced by expelled gases created either by igniting powder or causing the combustion of two liquids. The propulsion is produced by recoil due to the gases and not by a push against the air.

It is planned to shoot the rocket straight up into the air and to control its descent by means of a parachute. It will carry thermometers, barometers, electrical measuring apparatus, air traps to collect samples of air and other instruments to bring back much needed information.

The rockets used so far have been very small but experiments have been uniformly successful. No immediate experiments with large rockets are contemplated but eventually it is hoped that the rockets will bring back very valuable information about the air from 50 to 75 miles or more above the earth's surface.

R.

THE DIAMOND MINES OF SOUTH AFRICA.

BY

F. LYNWOOD GARRISON,

Member of the Institute.

Since the modern times the diamond has ever been the characteristic mineral of South Africa. Its discovery during the year 1867 in the alluvial river gravels of the Vaal Valley was a major event which gave an impetus to the settlement and development of this then remote section of the world greater than any subsequent occurrence in its history save that of the discovery of gold ten years later on the Witwatersrand where now stands the fine, large city of Johannesburg. It also affords an excellent example of the important rôle mineral discoveries, and subsequently developed mining industries, have played in the civilization and settlement by the white race in the wild sections of the earth.

It cannot be said the diamond is, or ever has been, of actual utilitarian value to mankind except, as it is the hardest known substance, for a few rather unimportant, or minor, purposes such as an abrasive and cutting material. Its æsthetic or decorative qualities and value appears to have been unknown to the ancients although they were familiar with such qualities in other gem stones. The beauties of the diamond were hidden and no doubt unsuspected until someone revealed them by grinding and polishing. The art of diamond cutting as this process is called, appears to have developed about the middle of the fifteenth century, although crude attempts to polish diamonds were practiced long before that time.

Prior to the discovery of diamonds in South Africa the sole sources of supply appears to have been the alluvial river gravels in certain sections of India and Brazil. This yield was never large as compared with later production in South Africa where diamonds are found not only abundantly in alluvials, but also in what might be called, colloquially but not technically, their mother rock.

The story of the African diamond discoveries and the early wild days of Kimberley and other "damond diggings" is much like that of our western mining camps before the industry settled down to an ordered and regulated business. It has been told many times in all its dramatic and picturesque phases which have little bearing on the purpose of this article save to observe that it was the commanding personality of Cecil Rhodes that brought order out of disorder, indiscriminate production discouraged and regulated; values stabilized and the whole industry placed on a sound, profitable basis.

Dry Screening. Consolidated Diamond Mines Ltd. West Coast.

In the early days, and even now, the discoveries of productive deposits and the production of diamonds in South and Central Africa has been considerably in excess of the demand with the result that at times the value of the product dropped almost, if not quite, to the cost of production and as the market for luxuries is necessarily limited, something had to be done to control and stabilize the production, This Rhodes accomplished and the results of his broad-minded policy are to be found today in the character and management of the DeBeers Consolidated Mines, Ltd., probably the best conducted mining organization in Africa. Indeed it might be said, Rhodes has been the most important man in the history of modern

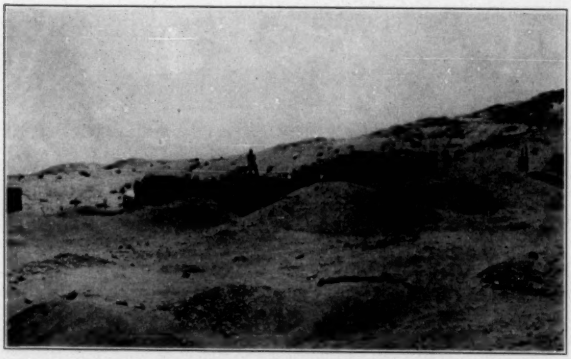
Africa; he not only placed the diamond industry on a solid basis, but also indirectly saved the gold mining industry of the Transvaal from strangulation by the conservative but doubtless well-meaning Boers and this gold industry has produced in the forty years of its existence over five billion dollars worth of this precious metal. An even greater achievement of Rhodes was his adding to the British Empire the vast and potentially rich territory now known as Rhodesia through the medium of the British South African Company commonly known as the "Charted Company."

The Old Kimberley Mine.

He defeated the astute Lobengula and pacified the warlike and virile Matabele (a branch of the Zulus) largely by the strength of his personality. Truly it is given to few men to accomplish so much by peaceful methods as did Rhodes in his short life of 49 years and to crown his great achievement with the Rhodes Oxford scholarships in a broad-minded effort to cement friendship between the two great English-speaking nations and their closely related Teutonic neighbors.

The mineral diamond is pure carbon, and carbon as we all know is the most abundant element in Nature. In its illimitable combinations it is the basis of all vegetable and animal substances as well as mineral oil, coal and natural gases. In

its pure condition it is exceedingly rare and occurs only in its crystalline form in what is known as the isometric system, the cube and the octahedron, the former shape being quite unusual and the latter relatively common. It is commonly pure white to the untrained eye, although not infrequently tinted various shades of yellow, pink, and blue. The bluish-white grades are considered the most valuable since they yield the best gems when cut. The real black diamond or "carbonado" is unfit for gems but being the hardest variety of diamond is used for drills and cutting tools. Carbons, or



Namaqua Diamond Company, Luderitz Bay, West Coast. Alluvial operation.

carbonado, are exceedingly rare, being found only with the alluvials of a few streams in the State of Bahia, Brazil and during recent years in small amounts in British Guiana. As far as the writer is aware this type of diamond has never been found in Africa which, if correct, is a remarkable fact, considering the relatively great abundance of all other forms of the mineral in this vast continent. It is only reasonable to expect they will sooner or later be discovered in Africa.

In the rough, that is uncut, diamonds resemble crystalline quartz or so called rock crystal. Both minerals will cut glass; their respective specific gravity, or weight, is much the same, being for diamonds 3.3 and for quartz 2.8. This resemblance

makes it difficult for the uninitiated to recognize a diamond when found on the surface of the ground or in gravel beds and is probably one of the reasons the ancients failed to recognize its value just as its presence in South Africa was for so long unsuspected by the Boer farmers upon whose lands in certain sections of that country it was by no means uncommon. The story of its ultimate discovery on these lands has been often told and is one of the most romantic chapters in the history of that country. Most people know gold by sight although there are certain easily identified metallic sulphides which are

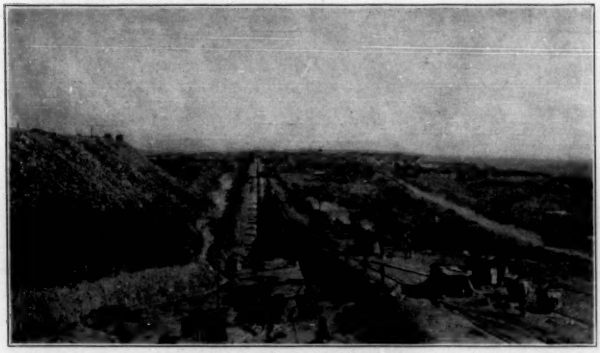
Namaqua Diamond Company, Luderitz Bay, West Coast. Stripping diamond bearing alluvials.

sometimes mistaken for it, very few would recognize a rough diamond when found associated with common white or iron-stained pebbles.

In South Africa diamonds were first found "in place" or in rock, in other words their place of origin, at the edge of the Karoo plateau where the city of Kimberley now stands, 647 miles north of Capetown. This was in the year 1870 which marks the beginning of the industry now sixty years old. Here the diamonds occur and were probably formed in a peculiar material called *kimberlite*, a basic or magnesian rock of volcanic origin. It is known to the miners as "blue earth or blue ground" and appears to be peculiar to Southern

Africa in its diamond-bearing character although there is no reason to suppose it may not ultimately be found in other parts of the world.

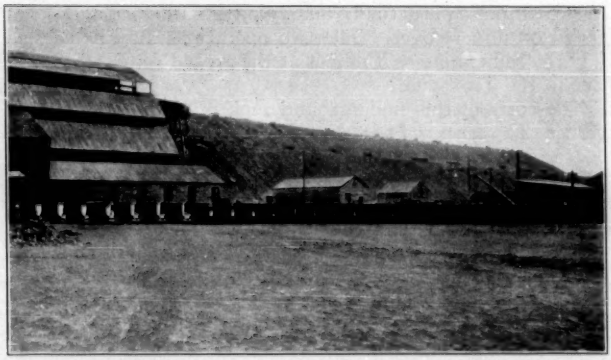
The genesis or origin of the diamond is one of the most obscure problems facing the geologist. It seems evident it is very closely connected with eruptive or volcanic phenomena. Kimberlite occurs in what was once considered volcanic plugs, or pipes of consolidated effusive masses of magmatic, molten rock which have been thrust up through faults or fissures in overlying sedimentary rocks and subsequently exposed to



Premier Mine. Waste dump.

view by the erosion and denudation of these superincumbent sediments. There are doubtlessi nstances yet to be found where such plugs have failed to reach near enough to the surface to be thus exposed. Herein lies the hope of finding new diamondiferous ground and the geologist has every reason to assume this expectation will be realized in the diamond bearing sections of Africa. As it now is over a hundred of these pipes have been discovered of which about twenty-five have been mined but only seven are actually producing diamonds. The rarity of these precious stones can be appreciated by the established fact that the proportion of gems to the amount of rock mined is something like 1 to 700,000 and sometimes 1 to

4,000,000. A better idea of this ratio can be had from the average production of two of the best mines in South Africa, the Premier near Pretoria, where the rate is seventeen carats to the hundred loads of rock mined, and the Wesselton mine in Kimberley, where it is twenty-three carats to the one hundred loads. A load is the capacity of a filled mine car of sixteen cubic feet. These carat rates naturally vary at different mines and is never uniform at any locality because the amount of diamond per cubic foot of rock (kimberlite) varies with the depth from the surface with a decided tendency to decrease



Wesselton Mine, Kimberley. Train of trucks carrying concentrates to the pulsators and grease tables.

with the depth at which it is mined. In this connection it is important to distinguish between rock mining and alluvial washing, or mining, and, in the former case, between underground, or undercover, mining and open-pit or, as it is technically called, open-cast mining. As these terms may be confusing it should be explained that underground mining is mining in its literal sense, that is operations away from daylight. Alluvial washings, or mining, as it is sometimes incorrectly called, is simply surface operations in relatively soft, unconsolidated material. The latter is a distinctly different source of diamond supply and will be mentioned later. In

the early days at Kimberley before the pits became too deep, all operations were open-cast and mostly individual, that is there were a large number of operators each having his own hoisting apparatus from a small block of ground independently worked. This caused not only great confusion and disputes but became increasingly dangerous with depth from the sides of the excavations falling in. With the consolidation of these interests this system of confusion was abandoned for normal underground operations by means of shafts which have now reached a depth of a thousand feet below the bottom of the old pits and about two thousand feet from the surface. Open-cast mining is actually quarrying of hard rock and is not carried on under cover. The two operations may be carried on in the same mine as it now is at the great Premier diamond mine in the Transvaal.

The costs of this underground rock mining works out an average of 83 cents to $1.18 per truck load or car and, taking the 100 load factor, we have $83.00 to $118.00 per 20 carats or $4.10 to $5.90 per carat. These figures are very sketchy or approximate and are stated solely to give a general conception of rock mining costs which usually is, but not necessarily, greater than alluvial mining, depending obviously upon local conditions. Other expenses, which doubtless vary considerably, must be added so it is difficult to determine what a general average cost of diamond production may be. The prices at which uncut diamonds are sold is however a matter of record and has varied during the past eighteen years between $8.00 and $11.00 per carat for "mine stones" and $20.00 to $27.00 for alluvial diamonds which for certain reasons are often regarded as the more valuable. It might be difficult to explain these reasons, if indeed they are actually understood; at any rate they are quite technical and for that reason cannot be discussed in an article of this character.

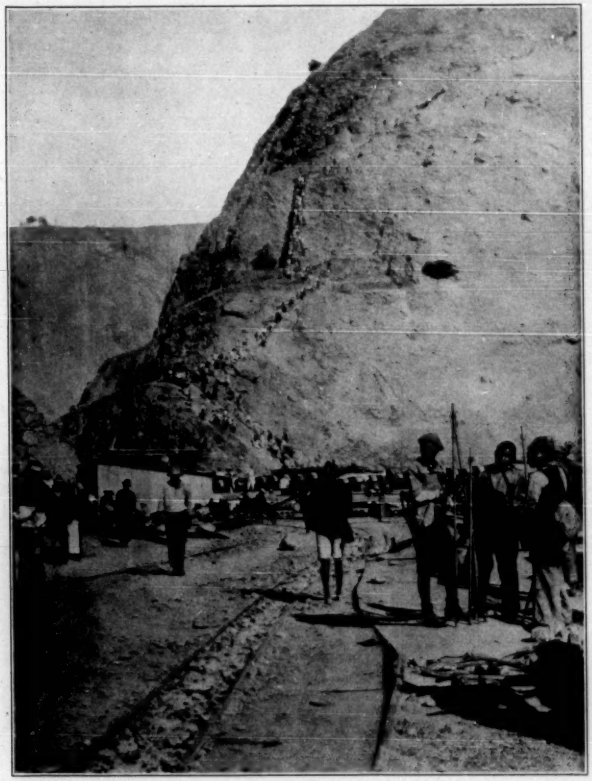
The South African Government imposes an export duty of 10 per cent. on all rough stones which with the inevitable cost and loss in grading and cutting must be added to the market value of the finished product. When the American purchaser is considering costs he should take into consideration in addition the import duty, the dealer's legitimate profit, the character, weight and color of the stones. This is perhaps a

rather complicated calculation and peculiar to the diamond dealer's trade, but the base estimates above stated may be useful to a purchaser in guessing or approximating market

Head frame, Wesselton shaft, Kimberley.

values. As is well known the diamond cutting industry is centered in Antwerp and Amsterdam and is a highly specialized art. Attempts have been made to establish it in South Africa with but indifferent success.

Rock diamonds are usually found in well-formed crystals and cleavage fragments (although the hardest known sub-



Premier Mine. Shift going out. Climbing up side of pit.

stance, diamonds are easily broken when struck along the cleavage planes). Alluvial stones usually exhibit evidences of

abrasion, their sharp angles being rounded off. The better class of diamonds is accompanied by "bort" and impure stones classed as "rubbish." Bort is impure or imperfectly crystallized diamond matter frequently occurring in globular or spherical masses with rough surfaces. Carbon or carbonado is sometimes confused with bort, but they are distinctly different phases of the same substance or element. The size of South African diamonds differs widely from the great Cullinan of 3,025¾ carats (1.37 lb. avoirdupois) to mere crystalline specks. The average size varies with the different localities where they occur, twenty-nine to sixty-five per cent. being over one carat (200 milligrams) in the more important mines. White stones tinged with yellow ("Capes and Silver Capes") are the most common and a very remarkable fact is that the product of each different mine or locality seems to possess a characteristic tint or tinge peculiar to itself so that experts can determine from what locality any particular stone may be derived. In fact it is claimed that every distinct and separate mass of diamond-bearing rock yields stones characterized by their own peculiarities of crystallization, color, luster, surface markings, etc. Furthermore it has been well established that in the pipes, or volcanic plugs, there is evidence of several distinct sections of the kimberlite which probably represent different successive periods, or phases of magmatic intrusion or inflow, and it is further claimed each section yields diamonds more or less characteristic of that section and can be thus differentiated or distinguished. It seems quite certain these pipes are made up of successive eruptions and the so-called "blue ground" thus formed in the same pipe possesses distinctly different characteristics. It has been proved these differences persist in depth and are probably due to variations in the nature of the kimberlite rock from which the blue ground has been derived by hydration or decomposition. It is consequently possible to divide the contents of some of the larger pipes into several distinct sections whose contacts in most instances are well defined. Each section being distinct it is assumed that the Kimberley pipe which has been the one longest worked and consequently best known, has been the product of at least three eruptions, or chimneys, which owe their origin to a progressive shifting of the center of

eruption along a west-northwest and east-southeast fissure. These pipes outcropping on a flat surface and being flush therewith, the diamond bearing material or blue-ground near the surface and hard kimberlite rock in depth, was quarried and dug out, transported to a suitable locality where machinery had been installed for crushing, screening and the recovery of the diamonds. As previously mentioned when the Kimberley pits became too deep for safety, underground methods had to be adopted, and at the Wesselton mine of this group it is estimated that, at present rate of production, there are enough reserves to last over ten or fifteen years.

At the Premier mine near Pretoria which is considerably younger and much larger than any of the Kimberley units, the open-cast methods continue in use but this enormous pit is becoming so deep another system will have to be adopted and preparations with that purpose in view are already being made. The great depth of about 1000 feet already attained at the open-cast Premier mine is well illustrated by the accompanying photographs recently taken by the writer. It was possible safely to reach this depth only because of the great size of the pipe and the strength of the walls. The depths attained by open-casting at Kimberley were between 400 and 500 feet and at the Jagersfontein 780 feet.

The important centers of rock diamond mining in South Africa are Kimberley, the Premier mine in the little town of Cullinan twenty miles east of Pretoria and at Jagersfontein in the Orange Free State. Although the cost of diamond production in these and other less important sections of the same character is greater than in the alluvials and can be more readily controlled, the enormous yield from the easily worked and extensive stream and bench gravels has of recent years tended to demoralize the market. To save the industry Governmental control became necessary. The Premier Diamond Mining Company has not paid a dividend on its common stock since 1927 and the New Jagersfontein as well as the DeBeers have had to decrease theirs. It has also been necessary for the large and strong companies to hold back from the market a considerable part of their production or else decrease their operations to a degree that might demoralize their organizations. The thousands of natives they have in compounds,

trained and indentured for definite periods of time cannot be laid off or dismissed and sent back to their usually rather distant homes without very serious consequences for the natives themselves, the companies and the country in general.

The diamond industry is peculiar in that its continued existence is dependent on the whims of fashion; if people ceased wearing diamonds for decorative purposes the industry

Premier Mine. Looking down into pit.

could not continue. Of course until human nature radically changes such a contingency is remote but the possibility must be admitted. On the other hand it seems unlikely that diamonds in any such quantity as occur in South Africa will ever be found in any other part of the world. Moreover the industry is soundly established in South Africa, is operated in a scientific, and skillful manner and if the almost overwhelming production from alluvials can be successfully restricted to meet the demand and not more, a fair profit to the operators is assured and the industry continued indefinitely.

Although it was in the alluvial gravels of the Vaal River

that the first discovery of diamonds was made in South Africa during the year 1867, and followed shortly afterwards by similar finds along the Orange River, these sources of supply were never as important as the rock mines later became. It has been only in the last few years that discoveries and production from alluvials in other districts have been so great as to threaten over production and disaster for the whole industry. At present the most important known detrital diamondiferous deposits are those of the Vaal and Harts Rivers, the deposits in the Lichtenburg area on the high plains of western Transvaal, those on the coast of Namaqualand both north and south of the mouth of the Orange River, and the beach or bench gravels on the sea-coast near Luderitz Bucht South West Africa. In the Lichtenburg area the diamond bearing gravels or "wash" have a very wide distribution but only part of the district has been worked. In South West Africa the original source of the diamonds now found in the alluvials is supposed to have been pipes of kimberlite not yet discovered or located, but probably some distance back in the interior from the coast. No doubt all the diamonds in the coast or sea beach deposits were brought down with the erosion detritus of the Orange River which was once much larger in volume than at present. This material was then distributed up and down the coast by ocean currents on beds and beaches now elevated considerable heights above the present tide level or beach line. Such vertical movements were certainly separated by great intervals of time, relatively short as geologists reckon it but enormously great when estimated in years.

These deposits are by their shallow, soft nature as compared with open cast and underground rock mining, very easily and cheaply worked. Some stripping of valueless overburden is usually necessary; the gravel is then dug out with pick and shovel or mechanical excavators. When cemented as such material often may be with some natural binder, blasting is necessary, followed by coarse crushing, screening and washing. The concentrate or fine material thus obtained is further cleaned in jigs, buddles and pulsators for the recovery of the diamonds. On the South West African coast which is an absolute desert, salt water is pumped from the sea for these operations which of course vary in completeness, design and

detail according to circumstances and the financial resources of the operator.

In the Lichtenburg district the amount of stripping necessary is large and as it is far from any flowing streams the water required for washing must be brought from a considerable distance at great expense. Moreover in this area operations relatively small and individualistic. As a consequence life in such a district is rough, crude and not conducive to settled, comfortable existence and those engaged in it roam from place to place as did the gold miners of California and Australia in the early days when the so called gold rushes developed. Such conditions of easily worked alluvial wealth are an irresistible lure to the adventurous, usually without substantial resources, or any knowledge of mining.

The coast or marine diamond deposits are very interesting, actually unique and have produced some remarkably fine stones. Thus there has been found a strip of gravel between eight and forty feet wide occupying an old storm beach raised from 110 to 135 feet above present sea-level which has produced diamonds of exquisite quality associated with almost no inferior stones or bort. A peculiar and remarkable fact about these high beach gravels is that the diamonds are accompanied with a characteristic fossil oyster (*ostrea prismatica*) a peculiar elongated bivalve that now lives only in the warm waters of the Indian Ocean hence it is believed that at the time of the formation of the high shingle or gravel deposits the coast line was sufficiently submerged to cause a northward flow up the west coast of the warm Mozambique current. The extent of some of these deposits is great, thus the mineral rights of the Consolidated Diamond Mines of South Africa the largest company operating in that region, extend from north of Luderitz to the north bank of the Orange River a distance of over 200 miles along the coast and 60 miles inland. A drawback to operations in this district is the absolutely arid and desert character of the land accompanied by enormous sand dunes hence the transportation of supplies and equipment is expensive and difficult. Operations for the recovery of diamonds from alluvials are relatively and normally inexpensive when the amount of overburden necessary to be removed is not very large.

The economic features, or factors, relating to the diamond mining industry are important and of great interest. As South Africa alone can produce more diamonds than the market will absorb, it has been necessary for the principal producers to agree to a restricted joint production. Periodical compacts have accordingly been made between the "London Diamond Syndicate" and the four largest producers in South Africa, the DeBeers; South West Africa; Premier; and Jagersfontein, whereby the "Syndicate" buys a definite quantity at an agreed price, each of these companies being allotted

Premier Mine. Waste dumps.

a certain proportion of the diamonds required for the market. During the year 1928 the total amount thus sold was valued at 4,574,969 pounds sterling for the Union of South Africa and 1,216,131 pounds for South West Africa, the latter coming practically wholly from alluvials. These two countries however no longer have a monopoly of the supply as other countries now contribute considerably to the demand, notably Angola (Portuguese), Belgium Congo, British Guiana and mandated South West Africa. These countries alone are said to be able to yield over one quarter of the world's production. Flooding the market from such sources has been in a measure checked by an agreement with the producers in

Angola and the Congo to restrict their output. Another even more difficult complication has of late years been caused by the large production of alluvial stones which are relatively more cheaply acquired and often, if not usually, of better quality than the diamonds from the rock mines; moreover this phase of the industry is more difficult to control.

Except in a few special cases all precious stones in South Africa are the property of the State and the Government retains the right to one-half if found on private, and seven-tenths when taken from Crown, reserves. Thus the State owns six-tenths interest in the great Premier Mine. Because of the ease with which diamonds can be stolen stringent laws have been enacted to prevent all trading in uncut diamonds except by properly licensed persons. All stones purchased have to be examined by Government experts who are usually able to distinguish the diamonds from various deposits as they appear to have as a rule their own peculiar characteristics, and these examinations serve as a check on the sale of stolen diamonds. Such regulations are especially necessary with the alluvial mines or diggings, as control in these cases is more difficult than with the rock mines. In Namaqualand the Government retains full possession of all diamonds on Crown reserves except where special discovery rights have been granted. Outside of such discovery claims all areas are operated as "State Diggings."

An export tax of 10 per cent. on uncut diamonds has been in force since 1919 and there is an additional tax of 15 per cent. on profits. Between the years 1911 and 1913 the average prices for so-called mine stones ranged between eight and ten dollars per metric carat of 200 milligrams. In 1920 this price average rose to twenty dollars and at the end of that year a complete collapse occurred in the diamond trade due to over production and a general world-wide financial depression. The prices of alluvial stones range much higher than those of mine stones, although they are equally affected in value in sympathy with market fluctuations. Average prices are also determined by the quality and size of the diamonds produced. The large quantity of stones of small size and comparatively poor quality sold from the Lichtenburg fields tended in 1928 to lower the value of the stones of good size and fine quality

from Namaqualand alluvial field. At present (1930) the average price for all stones is about from ten to twelve dollars per carat.

The production of alluvial stones could be vastly increased by a general use of mechanical methods such as are now being installed in the South West African field along the coast. The potentially productive areas in this section are said to be very large and the cost of operation considerably less than in the rock mines and probably also than in some of the alluvial districts. There can be no question that the diamond production of Africa in general could be increased far beyond the capacity of the market to absorb it now or probably for many years to come. Diamonds being what they are, a luxury and a non-essential, there would seem to be no moral or legitimate objection to a sufficient and regulated output so that the industry can be maintained and the capital and labor engaged in it have a fair return. It is generally agreed that the volcanic pipes alone in South Africa are capable of supplying the world's demand for an indefinite length of time in the future. Obviously the prosperity of the industry in South Africa will depend upon the continued control of production and a normal and contented supply of labor. The labor question in both South Africa and in Rhodesia is a very difficult one. People outside of these countries, except those who have had to deal with the question, imagine that such a thing cannot exist in Africa, that labor must be almost superabundant. A few observations on this head may not be without interest.

Africa below the tenth degree of south latitude, that is those sections where the mining industries are developing with increasing intensity, is under populated in not a few places. Hence for years it has been necessary to recruit natives from more thickly settled neighboring districts, especially the Portuguese territory on the East Coast. The reasons for this sparsity of population in certain sections and the difficulty the industries find in obtaining labor from other and usually rather distant regions are to be found in the history of the country, and the character and the habits of the natives themselves. It is important to remember we are dealing with a primitive people erstwhile savages whose wants are few, simple and easily supplied in a tropical or semi-

tropical country where, at present at any rate, there is plenty of land for everybody. Moreover distant employment from their homes means a greater or less separation from their families and these black people love their homes quite as much as we cherish ours. In the diamond mining districts such as Kimberley and in the great gold mines of the Witwatersrand about Johannesburg the natives employed in these industries are for the most part housed in compounds and separated from their families. Even with the best will and intentions this system is a difficult one both for the natives thus employed

Consolidated Diamond Mines, Ltd., West Coast. Alluvial washing plant.

and for the companies who operate the mines, and there is much to be said against it. On the other hand such objections are recognized and freely admitted, hence the companies and the governmental authorities do everything in their power to minimize the difficulties.

At the Premier diamond mine the average number of natives employed is slightly over 5,000 and their average wage is about 75 cents per day including piece work and overtime. At Kimberley the average wage is about 60 cents but they have certain compensating advantages not usual at other mining centers. The "head boys" at Kimberley and "drill runners" make from $1.15 to $1.25 per shift and the white employees

are well paid and treated with much consideration so that the white mining population becomes contented and settled. There is very little stealing of diamonds by the natives; when they find diamonds in the course of their work they usually turn them in and receive a reward of money which is more to their satisfaction than the risk of selling stones clandestinely, for to be possessed of an uncut diamond in South Africa is a misdemeanor. The compounds at Kimberley in which the natives voluntarily immune themselves under a four year indenture, are scrupulously clean, healthy and well conducted in every respect. The natives prepare their own food which is sold to them by the DeBeers Company at cost. As these natives come from a number of different tribes each with probably different customs, the system of allowing them to prepare their own food is a distinct advantage. They are also permitted and encouraged to hold religious services of any proper and decorous character they desire and are urged to teach themselves to read and write, which they appear very keen to do. The Kimberley compounds are large, with the best of sanitary equipment, planted with flower beds and supplied with an abundance of hot water for washing, with the result that a sense of beauty is developed along with habits of cleanliness which they take back to their homes when their term of employment ends. These influences for better living conditions are thus spread, and as the natives are maintained in good health in the mines and compounds, and are taught the advantages of sanitation and care of the body, they are likely to terminate these four years of service better in every respect than when they entered it. Moreover it is likely they will have saved a tidy sum of money for their families as may be seen by the following statement showing the disposal of moneys entered in the books of the Wesselton mine compound during the year ended June 30, 1929.

Taken by natives discharged from Compound. (Pounds sterling)	24,853 –9–9
Paid by natives to visiting relatives	3,754–17–6
Poll Tax paid to Registrar of natives	858 –5–0
Sent by Money Orders through Post Office	15,201–14–5
Postage stamps purchased by natives	1,030 –0–0
Registered envelopes purchased by natives	108 –5–0
	45,806–11–8
Deposited in Native Savings Account	10,526 –3–6
Deposited in Post Office Savings	1,245 –0–0

Probably other mine units of this great DeBeers Company would exhibit somewhat similar results although perhaps this is the best, as the Wesselton mine is also the most modern and efficiently equipped of all its properties. When it is considered that these natives receive, as a base wage, only two shillings and sixpence per shift, ranging up to five shillings as their skill and efficiency improves, it must be admitted this exhibit is a very hopeful indication of their thrift and desire for better living. No better proof of this could be asked than to point out that about $5,150 dollars was spent by them for

Consolidated Diamond Mines, Ltd., West Coast. Alluvial operations.

postage aside from Post Office money orders. In a very genuine sense this Wesselton Mine and Company is a model. The DeBeers is the oldest of the large African mining companies, it was founded by Cecil Rhodes and its present policies closely follow those he laid down at its inception. There is much to be said both for and against the compound system as it exists in South Africa. In the beginning it was necessary to prevent the theft of diamonds and to hold the natives to their work and perhaps to a smaller degree, this is still a valid reason for its existence. That it was subject to abuses in the early days is doubtless true but it has probably never deserved the harsh criticisms leveled at the system especially by

certain missionaries who seemingly never took the trouble to investigate for themselves. Both the companies and the Government have struggled with the problem. Much has been done to mitigate the admitted disadvantages of the system. The British the world over treat native populations with justice and kindness and in consequence have had comparatively little trouble. Africa has been no exception to this rule of conduct although in the past the Boers are said to have been harsh with the natives and the Belgians so cruel as to merit the

Wesselton Mine, Kimberley. Compound.

intervention of European powers some years ago and it is doubtful if they have changed.

The policy of the French in Northern Africa has been different and probably on the whole kindly. They appear to regard all young male natives as potential soldiers and are training thousands of them for modern warfare to supply possible needs in Europe. Most of the natives in French African possessions are Mohammedans and Islam is a notoriously militant creed and moreover is rapidly spreading in Africa north of the Equator.

However interesting the technical details, the history and the romantic side of the great diamond mining industry, it is even of more importance to study the effect of its development

upon a wild primitive country inhabited by savage negro tribes of diverse character controlled, or virtually ruled, by a small number of white people of Dutch origin who settled there as agriculturists and desired to be left alone to follow that vocation. An antipathetic state of mind was bound to arise in them towards its industrial development in other respects.

Before the discovery of diamonds in the Karoo the British element was small and is still relatively so. After that event there was a large influx of many sorts and conditions of people from all over the world, but largely British and not always of the best. This occasioned many difficulties and was the basal cause of the Anglo-Boer war which led eventually to the dominion status of the Union of South Africa, an act of liberal statesmanship difficult to match in history. The younger generation among the Dutch element certainly recognize this fact and also the advantages the industrial development of their country may afford them even although they may be at times reluctant to admit it.

The development of industry in South Africa requires labor in large quantities and the dominant sentiment everywhere in that vast country is rightly against the importation of Asiatics. The experiment was tried in Natal with Indians to provide labor for the sugar estates and Chinese were imported to labor in the Rand gold mines, but although they proved eminently satisfactory as miners, public sentiment both locally and in England proved inimical to their continued employment, and they had to be repatriated. The labor question is evidently the crux of the industrial situation in both the Union of South Africa and in Rhodesia; both countries are underpopulated and the perennial cry is for more native labor. The normal increase of the native population seems to be relatively small and, in the Portuguese sections, governed by such easy-going people, the natives are not encouraged to leave despite the higher wages and good care they unquestionably receive at the mines. The native labor in the Transvaal is recruited mainly from Zululand, Basotoland and Mozambique which has a combined native population of 3,500,000 from which it has been found difficult to obtain the 200,000 labor force necessary for the Transvaal alone. In Northern Rhodesia the progress of the development of the

vast copper deposits in that section will depend upon the supply of native labor, and there is little to be had locally, as this country has for many years been underpopulated. It has been estimated that the number of native men required at present for this development is not less than 40,000 which is probably much less than will be needed within the next few years. Where it is to come from no one seems to know. The importation of Asiatics will probably not be permitted.

Hotel Belgrave, Kimberley. Formerly the home of Cecil Rhodes. Now operated by the DeBeers Company.

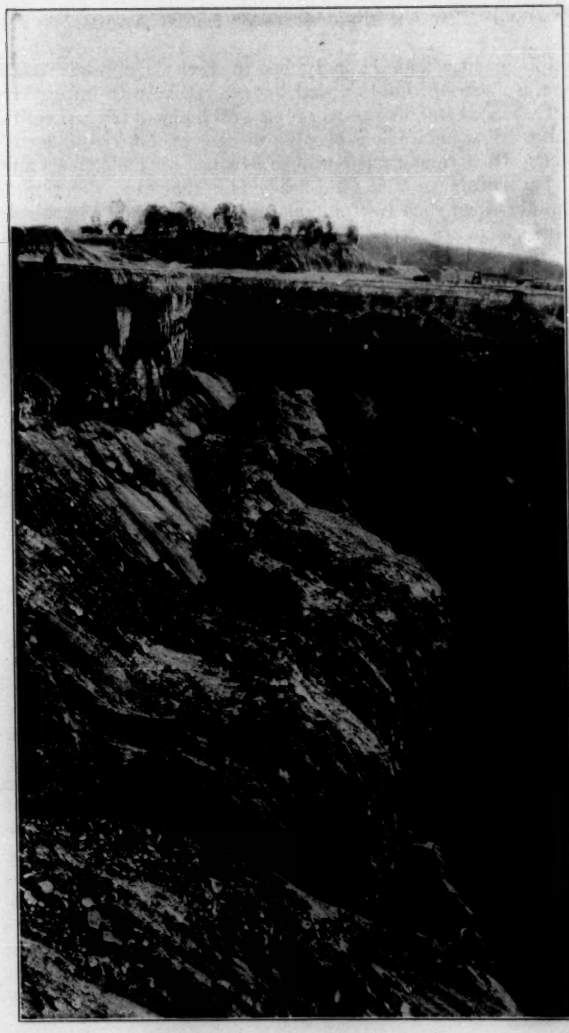
What may be the number of the present native population of Central and South Africa, there seems to be no way of determining. It is certain that within the past century or even the last sixty years whole sections of this part of the continent have been depopulated by the slave raids and the ruthless warfare waged against their fellow natives by such fierce tribes as the Zulu and Metabele. Great sections of what is now Rhodesia were thus almost completely desolated and reverted to a wilderness. The result has been that the present indigenous population is wholly inadequate to meet even the present demand for labor and as the mining and other industries develop and increase, the difficulty is certain to become accentuated.

The absence of adult males from their homes in distant

employment will increasingly tend to check the normal growth of the native population and in turn is likely to impair the native social stability and a proper development of their political or tribal institutions because the natives have been accustomed from time immemorial to be ruled by their chiefs and it has become a well established fact that they deteriorate rapidly when such restraint and direction becomes impaired. These facts are recognized by the white people of Africa that there evidently runs through the legislation of South Africa a disposition to place pressure on the native to seek productive employment. This is effected by various forms of taxes; in other words every effort is made to speed up the development of the immense potential resources of the country with a totally inadequate population for that purpose and a public sentiment wholly against the importation of Asiatic labor. Portuguese East Africa appears to be the reservoir from which the bulk of the present supply of native labor is drawn and the Portuguese, who were the first white settlers of Central and Southern Africa, are naturally adverse to this movement; in fact they have always been jealous of the British settling and developing the interior, which they could not, or would not, do for themselves.

The interior of South Africa below latitude ten degrees south is on the whole a healthy country, the high veldt, as it is called, ranging from two to six thousand feet above sea level. White children do fairly well on the whole for it is no hotter than Arizona, New Mexico and Southern California and with proper drainage and mosquito control ought to be equally healthy. In winter it gets rather cold and there are occasional snows as far north as Johannesburg.

With over-population and unemployment in England and also in Germany, the answer to the lack of labor in the healthy parts of South Africa seems obvious. It is true native labor is paid much less than white labor would demand; on the other hand white labor is more intelligent and consequently is likely to be more productive per unit of time. But it is doubtful, indeed highly improbable, that this white efficiency will necessarily offset its increased cost; many of the natives by no means lack intelligence, consequently they acquire skill and when healthy and well cared for, become efficient workers.



The old Kimberley pit, or pipe. 1200 feet in depth.

With a large influx of white unmarried males from Europe the question and problem of miscegenation must be faced. English sentiment is very strong on this question and outlaws a man who takes a native woman; this conviction colors the whole colonial policy and its enforcement is one of the reasons the British as a rule have been successful in governing the darker races. It is a well known fact that command of native troops, or authority in other respects, improves the Englishman or American and usually tends to debase the Latin. An English officer or official cannot command the respect if he takes a native wife; the Latin seldom has any such scruples and history has written large the result in South America and other countries.

White man's prestige in South Africa is or was largely due to the enormous impression Livingstone made with the natives. Stanley added to this superiority complex but did it in a different and perhaps less kindly manner. It is to be regretted this spiritual supremacy of the white man has been rather rudely shaken by the Great War and if it can ever be reasserted it will require years of patient, fair and tactful treatment. There is an awakened public conscience among the whites no less than with the natives which will not tolerate silently injustice or abuses towards the natives. The native needs the white man as much as, if not more than, the white man needs the native and it is unjust to ignore the proved capacity of the natives to improve their methods of production under proper guidance. Morevoer there are certain destructive tendencies among not a few of the native tribes that have resulted in their decrease or stationary numbers. This disposition or weakness can be doubtless overcome by medical attention and providing incentives to a fuller life. The industrialization of the country may accomplish this very desirable and laudable object but at the same time industrialization may carry in its train dangers to both black and white. It is desirable therefore that this development be not too fast nor pushed too feverishly as the pace now seems to be in the development of the copper resources of Northern Rhodesia. Here, enormous capital investments are being made to produce a metal with which the world is already well provided and result in an over-production everywhere disastrous. It seems

certain the increasing demand for labor in this section cannot be met which may prove the saving of the situation, thus halting development and checking the production of a commodity the world markets cannot now absorb except at a loss to the producer.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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GEAR TRAIN ON A ROTATING TRAIN (Fig. 80).

As an example of the dynamics of a mechanism on a rotating frame, we will consider the simple case of a gear train mounted completely on a rotating frame, F, that is, the bearings of the shafts of the gears and the motor housing, M', are attached rigidly to the rotating frame, F. The table is rotated by the meshing of the gear, G, with an inside circular rack fixed to the foundation, H. The motor, M, drives the gearing and sets up a reaction at the pitch point of the main gear, G, and rack, and thus rotates the table. For retarding the rotating system, a brake, B, on the shaft of the motor sets up a resisting torque.

With backlash in the teeth, particularly in the meshing of the main gear with the rack, severe stresses may be imposed on the teeth in the adjustment of this backlash with resulting impact for abrupt changes in acceleration to retardation. An obvious remedy is a torsional gear spring with damping in G.

Let us define the configuration of the gear train by the cyclic angular coördinate ϕ of the gear wheel, G, relative to the rotating frame, F. Also, let θ be the angular displacement of the rotating frame itself with respect to the foundation or stationary circular rack. The system has two degrees of freedom when not constrained by the rack. The condition equations of contact of the gear, G, with the rack during the

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 108, vol. 211, January, 1931.

acceleration and retardation period proper introduces a reaction of constraint and reduces the system to one degree of freedom. On the other hand, in the adjustment of tooth contact due to backlash in the teeth between the acceleration and retardation no reaction of constraint is offered by the teeth at point of contact of gear and rack, and we have a system of two degrees of freedom.

Let

M_n = the mass of any element of the gear train,

I_n = the mass moment of inertia of any element of the gear train,

I_f = the moment of inertia of the rotating frame not included in the above elements,

γ_n = the gear ratio to any element, i.e., $\frac{}{}$, where q is the relative angular displacement of the element,

r_n = the radius from the axis of rotation of the frame to the axis of any gear wheel attached to the frame,

R = radius of rack from central axis of table,

r = radius of main gear meshing with the rack,

M_t = the motor torque acting during the acceleration,

B_t = the brake torque acting during the retardation,

γ_t = motor or brake shaft gear ratio,

M_D = the friction reduced to an equivalent torque in the gearing,

M_D' = the friction torque at the pintle bearing of the frame,

γ_D = the corresponding gear ratio.

We have three periods to consider: (1) the acceleration interval, (2) the backlash interval with subsequent impact, and (3) the retardation period. In the interval immediately after acceleration, applying the brake torque causes a relative acceleration which sets up an impact velocity at the end of the backlash play, i.e., when the teeth are again brought into mesh for retardation.

(1) *Acceleration Period* (Fig. 81):

During this period we have a system with one degree of freedom, but we may consider the motion as of two degrees

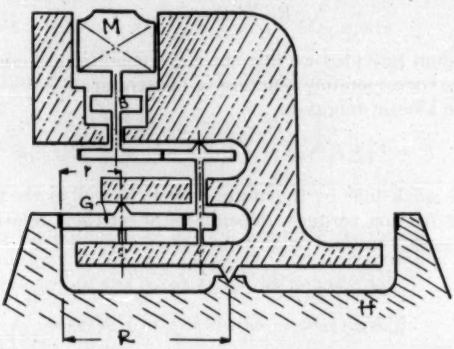

fig. 80

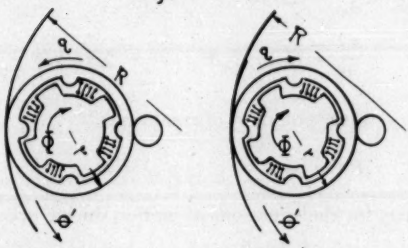

fig. 81

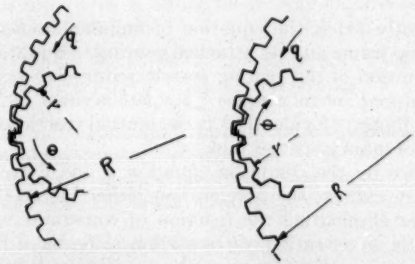

fig 82

of freedom provided we introduce the condition of constraint and the corresponding reaction of constraint.

The kinetic energy is

$$T = \tfrac{1}{2}\Sigma I_n(\gamma_n\dot{\phi} - \dot{\theta})^2 + \tfrac{1}{2}(I_f + \Sigma M_n r_n^2)\dot{\theta}^2.$$

The work done by the motor torque, as well as the internal gearing friction torques, depends only on their relative displacements, that is,

$$M_t(\gamma_t\delta\phi - \delta\theta) + M_t\delta\theta = M_t\gamma_t\delta\phi,$$
$$\Sigma M_d[(\gamma_d\delta\phi - \delta\theta) + \delta\theta] = \Sigma M_d\gamma_d\delta\phi.$$

Since

$$\frac{\partial T}{\partial\dot{\phi}} = \Sigma I_n(\gamma_n^2\dot{\phi} - \gamma_n\dot{\theta}),$$

$$\frac{\partial T}{\partial\dot{\theta}} = (I_f + \Sigma M_n r_n^2)\dot{\theta} - \Sigma I_n(\gamma_n\dot{\phi} - \dot{\theta}),$$

and noting the equation of condition,

$$R\dot{\theta} - r\dot{\phi} = 0; \qquad \therefore \quad \lambda(R\delta\theta - r\delta\phi) = 0,$$

we have, for the equations of motion during acceleration,

$$\Sigma I_n\gamma_n^2\ddot{\phi} - \Sigma I_n\gamma_n\ddot{\theta} = M_t\gamma_t - \Sigma M_d\gamma_d - \lambda r, \qquad (1)$$
$$(\Sigma I_n + I_f + \Sigma M_n r_n^2)\ddot{\theta} - \Sigma I_n\gamma_n\ddot{\phi} = \lambda R - M_d'. \qquad (2)$$

Evidently (2) is the equation of angular momentum of the rotating frame and its attached gearing. Equation (1) is the acceleration of the gearing system reduced to rest by adding the reduced inertia couples $\Sigma I_n\gamma_n\ddot{\theta}$ to account for the rotation of the frame. Evidently λ is the mutual reaction between the teeth of main gear and rack.

Since by the condition equation ϕ and θ are dependent, we may express the acceleration either in terms of ϕ or θ. Further eliminating the reaction of constraint we may estimate the acceleration of the system in terms of the acceleration torque. Choosing ϕ as the coördinate defining the con-

strained motion, we have, on eliminating λ,

$$\left[\Sigma I_n \left(\gamma_n - \frac{r}{R} \right)^2 + (I_f + \Sigma M_n r_n^2) \frac{r^2}{R^2} \right] \ddot{\phi} = M_t \gamma_t - \Sigma M_d \gamma_d - M_d' \frac{r}{R},$$

which is the equation of motion during acceleration.

To obtain the kinetic energy and the corresponding velocities at the end of the acceleration period, we have

$$\frac{1}{2}\left[\Sigma I_n \left(\gamma_n - \frac{r}{R} \right)^2 + (I_f + \Sigma M_n r_n^2) \frac{r^2}{R^2} \right] \dot{\phi}_a^2 = \int_0^{\phi_a} \left(M_t \gamma_t - \Sigma M_d \gamma_d - M_d' \frac{r}{R} \right) d\phi,$$

from which we can estimate the angular momentum about the pivot axis, that is,

$$\text{A.M.} = \left[(\Sigma I_n + I_f + \Sigma M_n r_n^2) \frac{r}{R} - \Sigma I_n \gamma_n \right] \dot{\phi}_a$$

at the end of the acceleration period.

(2) *Backlash Interval with Subsequent Impact*: At the end of the acceleration we now suppose the brake torque to be applied. For this interval, until the backlash play between the teeth is taken up, no external moment is exerted on the rotating table and, therefore, the rate of change of angular momentum is nil. This is also evident from a consideration of the Lagrangian equations.

As before the kinetic energy is

$$T = \tfrac{1}{2}\Sigma I_n (\gamma_n \dot{\phi} - \dot{\theta})^2 + \tfrac{1}{2}\Sigma M_n r_n^2 \dot{\theta}^2.$$

The work done by the brake torque, which depends only on its relative displacement, is $-B_t \gamma_t \delta\phi$.

The equations of motion are, neglecting the friction as small compared with the brake torque,

$$\Sigma I_n \gamma_n^2 \ddot{\phi} - \Sigma I_n \gamma_n \ddot{\theta} = -B_t \gamma_t, \qquad (3)$$

$$(\Sigma I_n + I_f + \Sigma M_n r_n^2) \ddot{\theta} - \Sigma I_n \gamma_n \ddot{\phi} = 0. \qquad (4)$$

Evidently (4) shows the rate of change of angular momentum about the pivot axis is nil. It is to be noted that the brake torque is an internal mutual reaction, so that its moment effect is nil when considering the entire rotating frame. From (4) we have

$$\ddot{\theta} = \frac{\sum I_n \gamma_n}{\sum I_n + I_f + \sum M_n r_n^2} \cdot \ddot{\phi}. \tag{5}$$

In the backlash period, from acceleration to retardation, subjected to the sudden application of the brake, we have a relative velocity between the teeth given by

$$R\dot{\theta} - r\dot{\phi} = S_r, \qquad \therefore \quad R\ddot{\theta} - r\ddot{\phi} = \ddot{S}_r;$$

$$\therefore \quad \left(\frac{R \sum I_n \gamma_n}{\sum I_n + I_f + \sum M_n r_n^2} - r \right) \ddot{\phi} = \ddot{S}_r.$$

But from the eqs. (3) and (5) we have

$$\left(\sum I_n \gamma_n^2 - \frac{(\sum I_n \gamma_n)^2}{\sum I_n + I_f + \sum M_n r_n^2} \right) \ddot{\phi} = - B_t \gamma_t,$$

so that the relative acceleration between gear and rack is

$$\dot{S}_r \frac{d\dot{S}_r}{dS_r} = \frac{B_t \gamma_t [r(\sum I_n + I_f + \sum M_n r_n^2) - R \sum I_n \gamma_n]}{\sum (I_n \gamma_n^2)(\sum I_n + I_f + \sum M_n r_n^2) - (\sum I_n r_n)^2}.$$

If ΔS_r is the total play between the teeth of rack and gear, the impact velocity is

$$\dot{S}_{ri} = \sqrt{\frac{2 B_t \gamma_t [r(\sum I_n + I_f + \sum M_n r_n^2) - R \sum I_n \gamma_n] \Delta S_r}{(\sum I_n \gamma_n^2)(\sum I_n + I_f + \sum M_n r_n^2) - (\sum I_n r_n)^2}}.$$

At impact, if we neglect the small deflection of the teeth, then for any finite displacement during the very short interval of impact we have $R\delta\theta - r\delta\phi = 0$, though the instantaneous velocity relations will vary. Further, the corresponding constraint reactions are now of an opposite character from that in the acceleration period so that we have

$$- \lambda(R\delta\theta - r\delta\phi) = 0.$$

The equations of motion during impact are

$$\sum I_n\gamma_n^2\Delta\phi - \sum I_n\gamma_n\Delta\theta = \lambda r,$$
$$(\sum I_n + I_f + \sum M_n r_n^2)\Delta\theta - \sum I_n\gamma_n\Delta\phi = -\lambda R,$$

where λr and $-\lambda R$ may now be regarded as the impulsive torques resulting from the impact reaction at the pitch point of rack and gear. The generalized equation of momentum becomes, if $\dot\phi_1$, $\dot\theta_1$ are the velocities before impact and $\dot\phi_2$, $\dot\theta_2$ are the velocities after impact,

$$(\sum I_n\gamma_n^2 R - \sum I_n\gamma_n r)(\dot\phi_2 - \dot\phi_1) = (\sum I_n\gamma_n^2 R - (\sum I_n + I_f + M_n r_n^2)r)(\dot\theta_2 - \dot\theta_1),$$

or

$$A(\dot\phi_2 - \dot\phi_1) = B(\dot\theta_2 - \dot\theta_1). \qquad (7)$$

The velocities $\dot\phi_1$ and $\dot\theta_1$ just before impact are obtained from the condition that the angular momentum just before impact must equal the angular momentum at the end of the acceleration period due to its being constant throughout the backlash period. That is,

$$(\sum I_n + I_f + \sum M_n r_n^2)\dot\theta_1 - \sum I_n\gamma_n\dot\phi_1 = \left((\sum I_n + I_f + \sum M_n r_n^2)\frac{r}{R} - \sum I_n\gamma_n\right)\dot\phi_a. \qquad (8)$$

We have, in addition, the kinematic relations before and after impact given by

$$R\dot\theta_1 - r\dot\phi_1 = \dot S_{ri} \quad (9) \qquad \text{and} \qquad R\dot\theta_2 - r\dot\phi_2 = 0, \quad (10)$$

were $\dot S_{ri}$ is determined from eq. (6).

The eqs. (7), (8), (9), and (10) give a solution for the determination of the velocities before and after impact.

To estimate the required resilience capacity ΔE required in the teeth or in a torsion spring in the gear we have

$$\Delta E = \leqq T_1 - T_2,$$

where

$$T_1 = \tfrac{1}{2}\sum I_n(\gamma_n\dot\phi_1 - \dot\theta_1)^2 + \tfrac{1}{2}(I_f + \sum M_n r_n^2)\dot\theta_1^2,$$
$$T_2 = \tfrac{1}{2}\sum I_n(\gamma_n\dot\phi_2 - \dot\theta_2)^2 + \tfrac{1}{2}(I_f + \sum M_n r_n^2)\dot\theta_2^2.$$

(3) *Retardation Period* (Fig. 81):

During the retardation, due to the constraint of the rack, the system is again brought to one degree of freedom. From the condition equation, $R\delta\theta - r\delta\phi = 0$, we obtain the reactions of the constraint acting on the coördinates θ and ϕ, that is,

$$-\lambda(R\delta\theta - r\delta\phi) = 0,$$

which are of an opposite character to that in the acceleration period.

In place of the positive work of the motor torque we have the negative work of the brake torque, $-B_t\gamma_t\delta\phi$ acting on the relative coördinate ϕ.

Therefore, the equations of motion are

$$\sum I_n\gamma_n^2\ddot{\phi} - \sum I_n\gamma_n\ddot{\theta} = -B_t\gamma_t - \sum M_d\gamma_d + \lambda r, \quad (11)$$

$$(\sum I_n + I_f + \sum M_n r_n^2)\ddot{\theta} - \sum I_n\gamma_n\ddot{\phi} = -\lambda R - M_d'. \quad (12)$$

Eliminating λ, and expressing $\ddot{\theta}$ in terms of $\ddot{\phi}$ from the condition relation, we have, for the equation of retardation,

$$\left[\sum I_n\left(\gamma_n - \frac{r}{R}\right)^2 + (I_f + \sum M_n r_n^2)\frac{r^2}{R^2}\right]\ddot{\phi} = -B_t\gamma_t - \sum M_d\gamma_d - M_d'\frac{r}{R}.$$

Oscillation of the System with a Flexible Gear.—In order to reduce the stresses in the teeth of rack and main gear we will suppose inserted a torsional spring connecting the spider of the main gear which is rigidly connected with the gear train, to a floating gear rim which engages in the stationary rack. Let (Figs. 80–82)

M_r and I_r = the mass and polar moment of inertia of the rim,

q = the relative angular coördinate for the relative angular displacement between the rim and spider of the gear,

C = the elastic torsional constant of the spring connecting spider and floating rim of gear.

The system now has three degrees of freedom but is reduced to two when contact takes place between gear and rack by

the corresponding condition equation. The condition equation during the acceleration period is

$$R\dot{\theta} - r\dot{\phi} + r\dot{q} = 0; \qquad \therefore \quad R\delta\theta - r\delta\phi + r\delta q = 0,$$

and during the retardation period, since the relative displacement q is reversed due to the change in contact of the teeth, is

$$R\dot{\theta} - r\dot{\phi} - r\dot{q} = 0; \qquad \therefore \quad R\delta\theta - r\delta\phi - r\delta q = 0.$$

Since during braking we have a reversal of contact at the rack, so that the constraint reactions are of an opposite character from the acceleration period, we have the constraint reactions related by the condition equations in the form

$$\lambda(R\delta\theta - r\delta\phi + r\delta q) = 0, \quad \text{for the acceleration.}$$
$$-\lambda(R\delta\theta - r\delta\phi - r\delta q) = 0, \quad \text{for the retardation.}$$

Therefore, the constraint reactions, corresponding to the coordinates θ, ϕ and q, for the acceleration period, are

$$\lambda R, \quad -\lambda r, \quad \text{and} \quad \lambda r,$$

and for the retardation period,

$$-\lambda R, \quad \lambda r, \quad \text{and} \quad \lambda r.$$

The potential energy due to the torsional spring is

$$V = \tfrac{1}{2}Cq^2.$$

Also, noting the opposite character of q for the acceleration and retardation due to the reversal of contact of the teeth of gear and rack, the respective kinetic energies of the system for the acceleration and retardation period, are

$$T_a = \tfrac{1}{2}\Sigma I_n(\gamma_n\dot{\phi} - \dot{\theta})^2 + \tfrac{1}{2}(I_f + \Sigma M_n r_n^2)\dot{\theta}^2 + \tfrac{1}{2}I_r(\dot{\phi} - \dot{q} - \dot{\theta})^2,$$

$$T_r = \tfrac{1}{2}\Sigma I_n(\gamma_n\dot{\phi} - \dot{\theta})^2 + \tfrac{1}{2}(I_f + \Sigma M_n r_n^2)\dot{\theta}^2 + \tfrac{1}{2}I_r(\dot{\phi} + \dot{q} - \dot{\theta})^2,$$

where in $\Sigma M_n r_n^2$ the additional term for the rim is included but not in the term ΣI_n.

The equations of motion during the acceleration are

$$\sum I_n(\gamma_n{}^2\ddot{\phi} - \gamma_n\ddot{\theta}) + I_r(\ddot{\phi} - \ddot{q} - \ddot{\theta}) = M_t\gamma_t - \sum M_d\gamma_d - \lambda r, \quad (1)$$

$$(I_f + \sum I_n + \sum M_n r_n{}^2)\ddot{\theta} - \sum I_n\gamma_n\ddot{\phi} - I_r(\ddot{\phi} - \ddot{q} - \ddot{\theta}) = \lambda R - M_d', \quad (2)$$

and

$$-I_r(\phi - \ddot{q} - \ddot{\theta}) = -Cq + \lambda r. \quad (3)$$

Combining (1) and (3), we obtain

$$\sum I_n(\gamma_n{}^2\ddot{\phi} - \gamma_n\ddot{\theta}) = M_t\gamma_t - \sum M_d\gamma_d - Cq, \quad (4)$$

and from the condition equation, eliminating $\ddot{\theta}$, we have

$$\sum I_n\gamma_n\left[\left(\gamma_n - \frac{r}{R}\right)\ddot{\phi} + \frac{r}{R}\ddot{q}\right] = M_t\gamma_t - \sum M_d\gamma_d - Cq. \quad (5)$$

Eliminating λ from (1) and (2) and substituting for $\ddot{\theta}$ from the condition relation, we obtain

$$\left(\sum I_n\left(\gamma_n{}^2\frac{R}{r} - \gamma_n\right) + I_r\frac{(R-r)^2}{Rr} + (I_f + \sum I_n + \sum M_n r_n{}^2)\frac{r}{R} - \sum I_n\gamma_n\right)\ddot{\phi} + \left(\sum I_n\gamma_n - I_r\frac{(R-r)^2}{Rr} - (I_f + \sum I_n + \sum M_n r_n{}^2)\frac{r}{R}\right)\ddot{q} = M_t\gamma_t\frac{R}{r} - \sum M_d\gamma_d\frac{R}{r} - M_d'. \quad (6)$$

Combining (5) and (6), we obtain the equation of oscillation during the acceleration.

Since I_r is small, from eq. (3) we have $Cq = \lambda r$ (approximately) and eqs. (4) and (1) are the same. The eqs. (1) and (2) then become identical to the corresponding equations without the spring system.

For the retardation, the equations of motion are

$$\sum I_n(\gamma_n{}^2\ddot{\phi} - \gamma_n\ddot{\theta}) + I_r(\ddot{\phi} + \ddot{q} - \ddot{\theta}) = -B_t\gamma_t - \sum M_d\gamma_d + \lambda r, \quad (7)$$

$$(I_f + \sum I_n + \sum M_n r_n{}^2)\ddot{\theta} - \sum I_n\gamma_n\ddot{\phi} - I_r(\ddot{\phi} + q - \ddot{\theta}) = -\lambda R - M_d', \quad (8)$$

and

$$I_r(\ddot{\phi} + \ddot{q} - \ddot{\theta}) = - Cq + \lambda r. \qquad (9)$$

Combining (7) and (9), we obtain

$$\Sigma I_n(\gamma_n{}^2\ddot{\phi} - \gamma_n\ddot{\theta}) = - B_t\gamma_t - \Sigma M_d\gamma_d + Cq, \qquad (10)$$

and from the condition equation, eliminating $\ddot{\theta}$,

$$\Sigma I_n\left(\gamma_n{}^2 - \gamma_n\frac{r}{R}\right)\ddot{\phi} - \Sigma I_n\gamma_n\frac{r}{R}\ddot{q} = - B_t\gamma_t - \Sigma M_d\gamma_d + Cq. \qquad (11)$$

Again eliminating λ from (7) and (8) and substituting for $\ddot{\theta}$, we obtain

$$\left(\Sigma I_n\left(\gamma_n{}^2\frac{R}{r} - \gamma_n\right) + I_r\frac{(R-r)^2}{Rr} + (I_f + \Sigma I_n + \Sigma M_n r_n{}^2)\frac{r}{R} - \Sigma I_n\gamma_n\right)\ddot{\phi} + \left(I_r\frac{(R-r)^2}{Rr} - \Sigma I_n\gamma_n + (I_f + \Sigma I_n + \Sigma M_n r_n{}^2)\frac{r}{R}\right)\ddot{q} = - B_t\gamma_t\frac{R}{r} - \Sigma M_d\gamma_d\frac{R}{r} - M_d'. \qquad (12)$$

Combining (11) and (12), we obtain the equation of oscillation during the retardation.

Since I_r is small we have $Cq = \lambda r$ (approximately) and eqs. (7) and (10) then become the same. The eqs. (7) and (8) are, therefore, identical to the equations of retardation without the spring system.

During the backlash we have no contact of the teeth between the gear and rack, so that during this interval the system has three degrees of freedom.

Neglecting the small friction torque at the pivot of the rotating frame, we have the condition that the total angular momentum of the system about the pivot axis must remain constant.

Assuming the positive direction of the relative coördinate q the same as for the acceleration, then the kinetic energy of

the system is

$$T = \tfrac{1}{2}\Sigma I_n(\gamma_n\dot{\phi} - \dot{\theta})^2 + \tfrac{1}{2}(I_f + \Sigma M_n r_n^2)\dot{\theta}^2 + \tfrac{1}{2}I_r(\dot{\phi} - \dot{q} - \dot{\theta})^2,$$

where in $\Sigma M_n r_n^2$ the additional term for the rim is included but not in the term ΣI_n. The potential energy is $V = \tfrac{1}{2}Cq^2$. The system has no external constraint and, therefore, has three degrees of freedom.

The reduced forces acting on the coördinate ϕ are $-B_t\gamma_t$ and $-\Sigma M_d\gamma_d$. As all the rotational elements have the common rotation of the frame $\delta\theta$, the work of the mutual reactions between the parts on the coördinate $\delta\theta$ are nil. This applies, of course, to the friction and brake torques, etc. We neglect the external friction torque M_d' at the pivot axis of the frame. The elastic torque between the gear rim and spider acting on the coördinate δq is $-Cq$. This latter, however, is accounted for in the elastic potential function.

The equations of motion are

$$\Sigma I_n\gamma_n^2\ddot{\phi} - \Sigma I_n\gamma_n\ddot{\theta} + I_r(\ddot{\phi} - \ddot{q} - \ddot{\theta}) = -B_t\gamma_t - \Sigma M_d\gamma_d, \quad (1)$$

$$(\Sigma I_n + I_f + \Sigma M_n r_n^2)\ddot{\theta} - \Sigma I_n\gamma_n\ddot{\phi} - I_r(\ddot{\phi} - \ddot{q} - \ddot{\theta}) = 0, \quad (2)$$

and

$$I_r(\ddot{\phi} - \ddot{q} - \ddot{\theta}) = Cq. \quad (3)$$

Equation (2) evidently shows the constancy of the angular momentum of the system. If we neglect the inertia terms containing I_r in (1) and (2), we have, on combining (1) and (2) with (3),

$$I_r\ddot{q} + Cq = -\left(\frac{AD - BC}{BD}\right)(B_t\gamma_t + M_d\gamma_d), \quad (4)$$

where

$$A = \Sigma I_n + I_f + \Sigma M_n r_n^2, \qquad B = \Sigma I_n\gamma_n^2 - (\Sigma I_n\gamma_n)^2,$$
$$C = \Sigma I_n\gamma_n, \quad D = \Sigma I_n\gamma_n^2(\Sigma I_n + I_f + \Sigma M_n r_n^2) - (\Sigma I_n\gamma_n)^2.$$

This shows, during the backlash period, a relative oscillation to take place between the rim and spider of the gear of frequency

$$\frac{1}{2\pi}\sqrt{\frac{C}{I_r}}.$$

GOVERNOR CONTROL.

A centrifugal governor has two degrees of freedom, the angular coördinate $\gamma\phi$ about the spindle axis, and the angular coördinate θ for the configuration of the mechanism with respect to the spindle axis. The coördinate ϕ also defines the configuration of the machine controlled by the governor, where γ is the constant gear ratio to the governor spindle. On the other hand, the coördinate θ is related with the driving torque of the machines by controlling the steam or gas supply.

Let

I = the inertia coefficient of the driven machine controlled by the governor,

B = the inertia coefficient of the governor mechanism about its spindle axis,

A = the inertia coefficient of the governor mechanism with respect to the coördinate θ,

$F = \frac{1}{2}k\dot{\theta}^2$ = the damping function of the governor mechism,

V = the total potential energy of the governor mechanism including the action of gravity, springs, etc.,

Φ = the excess driving torque over resisting torque acting on the engine, or machine.

The kinetic energy of the system is

$$T = \tfrac{1}{2}(I + \gamma^2 B)\dot{\phi}^2 + \tfrac{1}{2}A\dot{\theta}^2,$$

where it is to be particularly noted that both A and B are functions of θ corresponding to the configuration of the governor mechanism.

Now through the control valve, Φ is a function of the coordinate θ of the governor mechanism, that is,

$$\Phi = f(\theta), \qquad \text{and} \qquad -\frac{\partial\Phi}{\partial\theta}(\theta - \alpha)$$

is the change in torque over its equilibrium value, which latter corresponds to $\theta = \alpha$ of the governor mechanism.

The equations of motion are

$$\frac{d}{dt}((I+\gamma^2 B)\dot{\phi}) = \Phi,$$

$$\frac{d}{dt}(A\dot{\theta}) - \tfrac{1}{2}\frac{\partial A}{\partial \theta}\dot{\theta}^2 - \tfrac{1}{2}\gamma^2\frac{\partial B}{\partial \theta}\cdot\dot{\phi}^2 = -\frac{\partial V}{\partial \theta} - k\dot{\theta},$$

where the equilibrium value corresponds to

$$\tfrac{1}{2}\gamma^2\left(\frac{\partial B}{\partial \theta}\right)_{\theta=\alpha}\omega^2 = \left(\frac{\partial V}{\partial \theta}\right)_{\theta=\alpha}.$$

To study the nature of the oscillations, let

$$\theta - \alpha = \xi \qquad \text{and} \qquad \dot{\phi} = \omega + \dot{\eta}.$$

Substituting in the equations of motion, the equations of oscillation are

$$(I+\gamma^2 B)\eta + \gamma^2\frac{\partial B}{\partial \theta}\omega\xi = -\frac{\partial \Phi}{\partial \theta}\xi \qquad (1)$$

and

$$A\ddot{\xi} + \tfrac{1}{2}\frac{\partial A}{\partial \theta}\dot{\xi}^2 - \tfrac{1}{2}\gamma^2\frac{\partial B}{\partial \theta}(\omega^2 + 2\omega\dot{\eta}) = -\frac{\partial V}{\partial \theta} - k\dot{\xi}, \quad (2)$$

but we note

$$\frac{\partial V}{\partial \theta} = \left(\frac{\partial V}{\partial \theta}\right)_{\theta=\alpha} + \frac{\partial^2 V}{\partial \theta^2}\xi \qquad \text{and} \qquad \frac{\partial B}{\partial \theta} = \left(\frac{\partial B}{\partial \theta}\right)_{\theta=\alpha} + \frac{\partial^2 B}{\partial \theta^2}\xi,$$

so that the second equation reduces to

$$A\ddot{\xi} - \tfrac{1}{2}\gamma^2\frac{\partial^2 B}{\partial \theta^2}\omega^2\xi - \gamma^2\frac{\partial B}{\partial \theta}\omega\dot{\eta} = -\frac{\partial^2 V}{\partial \theta^2}\xi - k\dot{\xi}, \qquad (2')$$

where the second order terms of ξ and $\dot{\eta}$ are omitted and the equilibrium values cancel out, and the partial derivatives are taken at the configuration corresponding to the equilibrium value, i.e., $\theta = \alpha$.

Differentiating the second equation and then combining with the first, we obtain

$$A(I+\gamma^2 B)\dddot{\xi} + k\ddot{\xi} + \left[(I+\gamma^2 B)\left(\frac{\partial^2 V}{\partial \theta^2} - \tfrac{1}{2}\gamma^2\frac{\partial^2 B}{\partial \theta^2}\cdot\omega^2\right) + \gamma^4\left(\frac{\partial B}{\partial \theta}\right)^2\omega^2\right]\dot{\xi} + \gamma^2\frac{\partial B}{\partial \theta}\cdot\frac{\partial \Phi}{\partial \theta}\xi = 0,$$

which is the equation of oscillation of the system.

The criterion of stability requires the static loading due to the weights or spring reaction to exceed the centrifugal effect for small displacements from the equilibrium position, that is,

$$\delta\left(\frac{\partial V}{\partial\theta}\right)_{\theta=\alpha} > \tfrac{1}{2}\gamma^2\delta\left(\frac{\partial B}{\partial\theta}\right)_{\theta=\alpha}\omega^2,$$

so that, for stability,

$$\frac{\partial^2 V}{\partial\theta^2} - \tfrac{1}{2}\gamma^2\frac{\partial^2 B}{\partial\theta^2} > 0.$$

The equation of oscillation has therefore positive coefficients. If we assume ξ and η to vary as $e^{\lambda t}$, then

$$a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0,$$

where $a_1 = A(I + \gamma^2 B)$, $a_2 = k$, etc. We may write this in the simple cubic form

$$x^3 + ax^2 + bx + c = 0,$$

where

$$a = \frac{a_2}{a_1}, \qquad b = \frac{a_3}{a_1}, \qquad c = \frac{a_4}{a_1}.$$

The solution of this cubic gives one negative and two complex roots with real parts negative when $ab > c$ and the real parts positive when $ab < c$.

To appreciate the significance of the character of the roots in regard to stability we note the solution for the equation of oscillation is

$$\xi = A_1e^{\lambda_1 t} + A_2e^{\lambda_2 t} + A_3e^{\lambda_3 t}.$$

et $\lambda_1 = -r$ for the negative real root, and

$$\lambda_2 = p + iq \quad \text{and} \quad \lambda_3 = p - iq,$$

so that

$$\xi = A_1e^{-rt} + Be^{pt}\cos(qt + L).$$

If the real part p of the complex root is positive, we have $ab < c$ and the oscillations continually increases. For stability therefore it is necessary that p be negative, that is,

$$ab > c \quad \text{or} \quad a_2a_3 > a_1a_4.$$

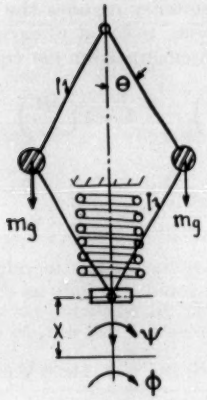

fig. 83

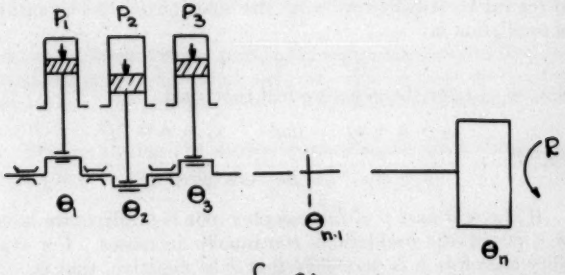

fig. 84

ANALYSIS OF GOVERNOR MECHANISMS (Fig. 83).

Let T = total kinetic energy of the mechanism,
m = mass of either ball,
Q = mass of the weighted sleeve,
C = the spring constant of the spring reacting on the sleeve,
x = the compression displacement along the spindle axis of the spring,
θ = the angular coördinate of the arms,
l = the length of the arms assumed equal,
$\psi = r\phi$, the angular coördinate about the spindle axis,
L = the corresponding torque of the spindle shaft.

The kinetic energy of the mechanism is

$$T = (ml^2 + 2Ql^2 \sin^2\theta)\dot{\theta}^2 + ml^2 \sin^2\theta\dot{\psi}^2,$$

where we note

$$A = 2(ml^2 + 2Ql^2 \sin^2\theta), \qquad B = 2ml^2 \sin^2\theta.$$

For the potential energy we have

$$\delta V = -2mg\delta(l\cos\theta) - Qg\delta(2l\cos\theta) - C\cdot 2l(1-\cos\theta)\delta(2l\cos\theta)$$

so that

$$V = -2mgl\cos\theta - 2Qgl\cos\theta + 2Cl^2(1-\cos\theta)^2,$$

where we note, if $x = 2l(1-\cos\theta)$, the elastic potential is

$$\tfrac{1}{2}Cx^2 = 2Cl^2(1-\cos\theta)^2.$$

We will assume, as before, the friction allowed for by a dissipation function in the form $\frac{1}{2}K\dot{\theta}^2$. If, however, the friction is assumed as due to the sliding at the sleeve, then

$$F = \tfrac{1}{2}k\dot{x}^2 = 2kl^2\sin^2\theta\cdot\dot{\theta}^2,$$

so that the reduced force of friction is

$$-\frac{\partial F}{\partial\dot{\theta}} = 4kl^2\sin^2\theta\cdot\dot{\theta}.$$

This also could be obtained directly, since

$$-k\dot{x}\delta x = -k\cdot 2l\sin\theta\cdot\dot{\theta}\cdot 2l\sin\theta\cdot\delta\theta,$$

$$-\frac{\partial F}{\partial\dot{\theta}} = -4kl^2\sin^2\theta\cdot\dot{\theta}, \qquad \therefore \quad K = 4kl^2\sin^2\theta.$$

The equations of motion are

$$\begin{aligned} 2(ml^2 + 2Ql^2\sin^2\theta)\ddot{\theta} + 2Ql^2\sin 2\theta\cdot\dot{\theta}^2 - ml^2\sin 2\theta\cdot\dot{\psi}^2 \\ = -2(m+Q)gl\sin\theta - 4Cl^2(1-\cos\theta)\sin\theta - K\dot{\theta}, \\ 2ml^2\sin^2\theta\cdot\ddot{\psi} + 4ml^2\sin\theta\cos\theta\dot{\theta}\dot{\psi} = L. \end{aligned}$$

The last equation is the torque on the spindle shaft equal to the rate of change of the angular momentum of the mechanism about the spindle axis, that is,

$$L = \frac{d(\text{A.M.})}{dt} = \frac{d(2ml^2\sin^2\theta\dot{\psi})}{dt}.$$

The first equation can be verified directly by the application of the equation of energy together with that for the angular momentum. We have

$$L\dot{\psi} = \frac{dT}{dt} + \frac{dV}{dt} + \frac{dF}{d\dot{\theta}}\cdot\dot{\theta},$$

which on substituting for L given above and simplifying we obtain the first equation of motion.

We have seen the equations of oscillation can be established either by a direct application of the Lagrangian equations or by a combination of the equation of energy and the equation of angular momentum about the spindle axis.

On the other hand, we may proceed by the use of D'Alembert's equation, choosing our "virtual" displacements consistent with the time coördinate $\psi = \omega t$ constant, provided we introduce the actual kinetic reactions, which, of course, include the centrifugal forces due to the rotation about the spindle axis. In other words, we have a rotating frame due to the angular velocity about the spindle, and we choose our virtual displacements relative to this plant at time "t." We must then include in the virtual work, the kinetic reactions, which include the centrifugal force due to the rotation of the frame.

(*To be continued*)

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

STRENGTH OF MASONRY OF HOLLOW UNITS.

With the object of evaluating some of the factors affecting the strength of masonry a study has been made of the data on the compressive strength of large masonry specimens of hollow clay units reported in University of Illinois Bulletin No. 27 and Bureau of Standards Technologic Papers Nos. 238 and 311. The data in these publications were selected because the masonry specimens were similar in size and in the arrangement or bonding of the units to ordinary building walls. Moreover these reports contain rather complete data on physical tests of the materials and descriptions of the method of construction and workmanship. They appear to be the only published reports on tests of large masonry specimens of hollow masonry units in which as complete information on materials and workmanship is given.

Insufficient data are available for a systematic study to determine the relations between the many factors expected to influence the strength of masonry of hollow units. However, the results of the tests indicate that the strength of masonry may be predicted roughly from a knowledge of certain easily determined properties of the units and mortar materials.

The compressive strengths of walls of hollow units laid on end (with cells vertical) were approximately proportional to the proportion of the gross sectional areas of the units given a direct bearing at the mortar joints. This proportion depended upon the design of the units and the workmanship of the masons who constructed the specimens. Wall strengths were approximately proportional also to the one-half powers of the compressive strengths of the units (on gross area) and of the mortar specimens. These relations may be expressed by the following empirical equation

$$M = b\sqrt{um},$$

* Communicated by the Director.

where

M = compressive strength of end construction masonry, lbs./in². (gross area).

b = ratio of area in direct bearing at the bed joints to the gross area at the same planes.

u = compressive strength of the units, lbs./in². (gross area).

m = compressive strength of mortar specimens aged the same as the masonry, lbs./in².

These relations indicate that properties of the units other than their compressive strengths must be considered for even rough estimates of the compressive strength of end construction masonry. For example, if a mortar bearing is provided only for the exterior shells that are parallel with the exposed wall surfaces, they indicate that the strength of the masonry is approximately proportional to the thickness of those shells. At present the requirements of some specifications and building codes appear to be based upon the assumption that the compressive strength of the units alone is an adequate measure of the effect of the properties of units on the strength of end construction masonry. It seems from the test data that the effects of the geometrical properties of the units should be given consideration also.

The results of compressive tests of side construction masonry gives qualitative information, at least, showing that masonry strengths were dependent largely on the following factors: (1) mortar mixture, (2) compressive strength of units, (3) workmanship, and (4) ratio of thickness of bearing shells to span between vertical shells (or webs). There seemed to be no simple way of evaluating the effects of workmanship with side construction masonry as appeared to be possible for end construction masonry. The data indicate, however, that the following properties should be given consideration in the preparation of specifications for side construction units: (1) compressive strength, (2) ratio of thickness of bearing shells to span between vertical supports (shells and webs), and (3) features of design affecting the proportion of area given a bearing at bed joints.

Since the elastic properties and the designs of concrete units may differ widely from those of the clay units used in the

construction of the masonry specimens, the results may not be applicable directly to masonry of hollow concrete units.

SPALLING TESTS OF FIRE CLAY BRICK.

Two methods of testing the resistance of fire clay brick to thermal shock are in use at present. One of the methods was promulgated by Committee C–8 on Refractories of the American Society for Testing Materials and the other by the Federal Government. In some cases consumers request that the bricks under purchase comply with the requirements of the first test and in other cases with the requirements of the second. Such a situation is not altogether desirable because of the cost devolving on the manufacturer if he desires periodic reports giving the results of both tests. As a result it was decided to make a series of comparative thermal spalling tests to obtain data which would indicate, together with results obtained during the past four years in the bureau's general investigation of fire-clay refractories, whether it was desirable to make changes in the present Federal specifications for fire-clay brick.

Manufacturers furnished 22 different brands of fire-clay brick which were subjected to both the tentative test for thermal spalling of the American Society for Testing Materials, and also to the test as described in Federal specifications for fire-clay bricks. In addition the bricks were subjected to three modifications of these two tests. The maximum, minimum, and average number of heating and cooling cycles which the bricks withstood before failure, when tested by these five different methods, are given below, together with the nature of the test under which spalling was induced.

Method.	Number of Heating and Cooling Cycles before Failure.		
	Maximum.	Minimum.	Average.[1]
1	Less than 88	3	18
2	" " 66	11	34
3	88	13	31
4	19	2	7
5	Less than 100	10	34

[1] The averages were computed using 50 cycles as the maximum number to which any brand had been subjected.

The methods used corresponding with these values were as follows:

1. Tested in accordance with the requirements of the Federal Specifications.

2. Tested in accordance with the requirements of the Federal Specifications except the preliminary refiring at 1400° C. for five hours was omitted.

3. Tested according to the method prescribed by the American Society for Testing Materials.

4. Similar to method (3) except that pieces of the bricks which had become loosened from repeated heating and quenching were removed with the fingers instead of continuing the test until the loose pieces dropped off.

5. In this case one end of the bricks received a preliminary heat treatment at 1400° C. for five hours. They were then quenched from 850° C. in 2 inches of running water until the end reheated at 1400° C. had completely spalled off.

COÖPERATIVE STUDY OF LITHOGRAPHIC PAPERS.

A joint meeting of the Advisory Committee to the Bureau of Standards on Standardization of Lithographic Papers and the paper section of the bureau was held at the bureau on December 8. This committee was appointed by the Lithographic Technical Foundation, which requested research on this subject and which is coöperating in it. Results obtained to date were discussed and future work planned.

Considerable waste is said to be caused in lithographic printing processes through lack of knowledge of the properties that papers must possess to give the best results. It is hoped that the present research will develop this information which will then make possible the definite formulation of paper requirements.

The committee considered the results of the bureau's survey of 31 lithographic printing plants, based upon personal inspections in the plants and consultations with the operatives. These showed that misregister was the principal difficulty being experienced, although there was some trouble with lack of clearness of prints caused presumably by picking or linting of the paper, or chemical reactions from some of the paper

constituents. In view of these results it was agreed that causes of misregister should receive first consideration and and that clearness of prints would be taken up later. Some progress already made in study of causes of misregister, which is being carried on in a coöperating printing plant, was reported to the committee which assisted in planning details for the further work contemplated. This study involves a consideration of the effect of moisture, ink, and pressure relative to distortion of paper and consequent misregister.

The work planned relative to clearness of prints included laboratory and plant studies of the interrelation of printing inks and such paper properties as smoothness, porosity, tendency to pick, kind of sizing materials, and acidity or alkalinity.

In addition to the bureau representatives, the meeting was attended by the following:

R. F. Reed, director, department of lithographic research, University of Cincinnati, chairman, advisory committee.

Alfred B. Rode, president, Lithographic Technical Foundation, New York, N. Y.

W. R. Maull, vice president, Dill and Collins, Philadelphia, Pa.

Miss R. M. Cobb, chemist, Lowe Paper Company, Ridgefield, N. J.

B. L. Wehmhoff, technical director, Government Printing Office, Washington, D. C.

Dr. Hugo Stockmayer, chief chemist, General Printing Ink Corporation, Rutherford, N. J.

Wm. H. Merton, president, Strobridge Lithographing Company, Cincinnati, Ohio.

E. A. Sprong, production manager, American Lithograph Company, Buffalo, N. Y.

ADSORPTION OF ORGANIC SOLVENTS BY TEXTILE MATERIALS.

It has been observed that textile materials when rinsed in organic solvents such as ether, alcohol, chloroform, etc. prior to drying, retain some of the solvent even when they are dried at a temperature considerably above the boiling point of the solvent. In connection with a study of methods for the de-

termination of weighting on silk, a few tests were made to determine the amount of adsorbed solvent. It was found that when the silk is rinsed in alcohol and then in ether, it retains about 2 per cent. of its weight of the solvent after drying for 1½ hours in an air ventilated oven at temperatures between 105° and 100° C. If the material is then dipped in boiling water, hydroextracted and dried, the original dry weight is obtained to within 0.1 per cent.

RESISTANCE OF CHROMIUM-PLATED PLUG GAGES TO WEAR.

Plug gages which are used for measuring internal diameters as, for example, the diameter of the cylinders of automobile engines, are often plated with chromium. This use of chromium is based on the fact that it wears at a lower rate than most steels which are used for plug gages. There are, however, several types of chromium-plate which are produced by variations in the control of the electroplating bath. These types are spoken of in the industry as "bright," "milky," and "frosty" according to the appearance of the deposits.

A laboratory study has been made at the bureau of the wear-resistance of the three types of chromium deposits above described, these tests having been carried out on a special machine designed and built for this purpose. Tests were made on the "bright" deposits both as plated (unlapped) and also after lapping (polishing). The "milky" and "frosty" deposits were tested in the lapped condition only. Lapped chromium-plate was more resistant to wear than the unlapped plate.

In the process of the electrodeposition of chromium, hydrogen is evolved which to some extent is absorbed by the chromium-plate. Heating the chromium deposit to temperatures in the neighborhood of 300° C. was found to improve the quality of the chromium-plate, probably by driving out the hydrogen.

The "milky" and "frosty" deposits of chromium showed better performance in these tests than the "bright" deposits, excepting those of the latter type which had been heated to 300° C. When a thin coating of copper was deposited prior to plating with chromium, the plate exhibited greater resistance

to wear than a similar deposit of chromium applied without the copper.

The hardness of chromium deposited under the conditions above described was investigated. Photomicrographs of the surfaces of the chromium-plate showed the existence of cracks in the chromium deposits.

This investigation will be discussed at greater length in the February number of the *Bureau of Standards Journal of Research.*

MEASUREMENT OF PULSATING GAS FLOWS.

The minutes of the meeting of the main technical and research committee, natural gas department, American Gas Association, held in St. Louis December 2, 1930, contain the following statement relative to the measurement of natural gas:

"It was decided to instruct the Gas Measurement Committee to start at this time to investigate all available information concerning pulsation and its effects on gas measurement for the purpose of first developing methods of determining the presence and measuring the magnitude of vibrations in gas flow and later of minimizing such vibrations or otherwise making correct measurements."

This prescribed work may conveniently be divided into the following specific problems:

I. To develop some practical field method for detecting the presence of pulsations that are strong enough to cause appreciable errors in the indications of orifice meters.

II. To develop methods of minimizing the pulsations or their effects so that measurements by orifice meters will be as free as possible from errors due to such causes.

III. If possible to devise some practical method for measuring gas flows even when pulsations are present.

The personnel of the Gas Measurement Committee includes two representatives of the Bureau of Standards, Dr. Edgar Buckingham and H. S. Bean, chief of the bureau's gas measurement section and acting secretary of the committee. To provide the committee with as complete a background as possible of the work that has been done on pulsating gas flows,

and also to avoid duplication and unnecessary expense, Mr. Bean has mailed a questionnaire to gas companies, manufacturers, and engineering laboratories in which the following data are requested:

1. Have you made any tests of the effects of pulsation on the measurement of gas flows? If so, will you give a brief description of the tests, the set-up used, and results obtained?
2. Do you contemplate making any such tests in the immediate future? If so, are you willing to outline the proposed extent of such tests, set-up to be used, where and when the tests will be made?
3. Briefly relate any observations made in the course of routine plant operation of the effects of pulsations on the measurements of gas flow.
4. What suggestions can you offer the committee as to any work it may do on this problem, particularly in regard to:
   (*a*) conditions that should be covered;
   (*b*) tests that should be made, if any;
   (*c*) set-up to be used for such tests;
   (*d*) location of making such tests?
5. What facilities have you that could be used by the committee for making tests on this problem? Of particular interest will be
   (*a*) compressor stations with sections of the inlet and outlet lines which may be by-passed so as to permit the installation of various set-ups without interrupting plant operation;
   (*b*) small gas holders near (less than a mile from) a compressor station which could be used for actual measurement of the flow;
   (*c*) one or more compressors that may be operated on either air or gas independent of plant operation;
   (*d*) gas flows that are being blown to air that could be diverted through an experimental set-up;
   (*e*) plant and yard facilities that might be used for tests at night but which would not be available in the day time?

6. Would it be agreeable to you to have a member of the committee visit you for the purpose of
   (*a*) viewing the set-ups you have used or plan to use in your own tests;
   (*b*) discussing in detail with you the results of your tests or observations;
   (*c*) inspecting the facilities you offer the committee for making tests?

This questionnaire was discussed and approved at a meeting of the Gas Measurement Committee held at the Bureau of Standards on December 16. The program which will probably be followed in this investigation was also considered and an outline submitted by Mr. Bean was considered satisfactory. After the replies to the questionnaire have been received and carefully reviewed it is planned to have a member of the committee visit the companies giving the most helpful replies. Through such visits an accurate idea can be obtained of the data which these companies may have on hand and the suitability of the plant facilities for research work of this kind.

FIBER STRESSES OF WOOD POLES.

The American Standards Association, under date of November 28, 1930, approved the following standard values for ultimate fiber stresses of wood poles as used in electrical line construction.

Northern white cedar, 3600 pounds per square inch.
Western red cedar, 5600 " " " "
Chestnut, 6000 " " " "
Southern yellow pine (creosoted), 7400 pounds per square inch.

In the National Electrical Safety Code, rule 261A4(c) states "Tests are under way to determine ultimate stresses of woods and when values for ultimate stresses have been adopted as standard by the American Engineering Standards Committee (now the American Standards Association), the values thus determined shall be applied under this code and the values in Table 20 adjusted proportionately."

Allowable Fiber Stresses (in Pounds per Square Inch) for Wood Poles under Vertical and Transverse Loading.

	When Installed.									At Replacement.				
	Treated Poles. For Ultimate Fiber Stress of					Untreated Poles. For Ultimate Fiber Stress of				Treated or Untreated Poles. For Ultimate Fiber Stress of				
	7400	6000	5600	5000	3600	6000	5600	5000	3600	7400	6000	5600	5000	3600
At crossings:														
Poles in lines of one grade of construction throughout—														
Grade A	2470	2000	1870	1670	1200	2000	1870	1670	1200	3700	3000	2800	2500	1800
Grade B	3700	3000	2800	2500	1800	3000	2800	2500	1800	5550	4500	4200	3750	2700
Grade C	5550	4500	4200	3750	2700	4500	4200	3750	2700	11100	9000	8400	7500	5400
Poles in isolated sections of higher grade of construction in lines of a lower grade of construction—														
Grade A	2470	2000	1870	1670	1200	1500	1400	1250	900	3700	3000	2800	2500	1800
Grade B	3700	3000	2800	2500	1800	2000	1870	1670	1200	5550	4500	4200	3750	2700
Grade C	5550	4500	4200	3750	2700	3600	3360	3000	2160	11100	9000	8400	7500	5400
Elsewhere than at crossings:														
Grade A	2960	2400	2240	2000	1440	2000	1870	1670	1200	1440	3600	3360	3000	2160
Grade B	4440	3600	3360	3000	2160	3000	2800	2500	1800	7400	6000	5600	5000	3600
Grade C	7400	6000	5600	5000	3600	4500	4200	3750	2700	11100	9000	8400	7500	5400

The value approved by the American Standards Association for northern white cedar is the same as that already recognized in the code. The values for the three other woods will supersede those heretofore in use.

A revision of Table 20 containing the new values is given below and may be used in future applications of the National Electrical Safety Code.

RADIO METHOD FOR SYNCHRONIZING RECORDING APPARATUS.

A report which will appear in the February number of the Bureau of Standards Journal of Research describes a simple radio method used by the bureau for synchronizing radio fading recorders. It was necessary to have one of the recorders mounted in a laboratory automobile so that it could be moved to any desired distance from the fixed recorder at the control station. It was necessary that the recording tapes at the two different points run at exactly the same speed, since the changes to be studied were quite rapid and it was desired to superpose the records for comparison.

Each recorder was driven by a synchronous motor of the type used for clocks. The motor at the control station was connected directly to the 60-cycle power line. To this same source of power was connected a simple radio transmitter of the half-wave self-rectifying type giving a signal modulated at 60 cycles. This 60-cycle transmission was received at the portable station and amplified sufficiently to drive the synchronous motor which propelled the recorder there. It was found possible to control the portable recorder at a distance of 10 miles with lower power output from the transmitter.

The paper describes a method for marking the two records simultaneously so that they can be superposed.

Measure Cloud Height with New Instrument.—(U. S. Department of Agriculture—Clip Sheet 637.) Prof. C. F. Marvin, Chief of the Weather Bureau, United States Department of Agriculture, recently invented a simple instrument through which a weather observer can look at clouds over airports at night and determine their height, or the "height of ceiling," as the aviator terms it.

This instrument, named the clinometer, resembles a very short telescope, although it has no lenses, and is used in conjunction with a searchlight. The beams from the searchlight are projected on the bottom of a cloud, usually straight up. Five hundred feet from the searchlight—sometimes 1,000 feet—the observer stands and looks through the clinometer, ranging it on the light spot on the bottom of the cloud. A pendulum hanging straight down from the under side of the clinometer at all positions, is locked in position by a screw when the clinometer is properly aligned.

The pendulum shows on a graduated scale the exact angle at which the clinometer was held. A prepared chart in the hands of weather observers readily shows the height of clouds for each angle of the clinometer and corresponding to different distances of the searchlight.

Clinometers soon will be issued to all airports having Weather Bureau observers.

R.

ERRATUM.

REPORT ON THE WORK OF THE BARTOL RESEARCH FOUNDATION,

December 1930 Journal, page 689.

Fig. 12, page 712, the arrow at the top of the figure should point from left to right instead of right to left.

THE FRANKLIN INSTITUTE.

ANNUAL MEETING, WEDNESDAY, JANUARY 21, 1931.

The annual meeting of The Franklin Institute was called to order by the President, Mr. Hayward, at eight-fifteen p.m.

The President called for a reading of the minutes of the previous meeting. The Secretary announced that these minutes had been printed in full in the January number of the Journal of the Institute and moved that they be adopted as printed. The motion was seconded and carried unanimously.

When the President called for a report from the Tellers, the Secretary read the statement of the results of the balloting, and announced that the gentlemen whose names had been read had been duly elected and would constitute the officers of the Institute for the ensuing year, together with certain other gentlemen who had been elected in previous years for definite terms, as members of the Board of Managers.

President (to serve one year): NATHAN HAYWARD.

Vice-Presidents (to serve one year): HENRY HOWSON, C. C. TUTWILER, WALTON FORSTALL, W. CHATTIN WETHERILL.

Treasurer (to serve one year): BENJAMIN FRANKLIN.

Managers (to serve three years): F. T. CHAMBERS, C. H. K. CURTIS, S. S. FELS, R. W. LESLEY, MALCOLM LLOYD, JR., M. S. MORGAN, E. H. SANBORN, S. T. WAGNER. (To serve two years) A. ATWATER KENT. (To serve one year): L. H. KINNARD, ROBERT MCLEAN.

Mr. Hayward then resumed the Chair and expressed his thanks to the Committee of Tellers: Mr. W. N. Jennings, Dr. W. M. Stine, and Dr. T. K. Cleveland, for their services in the conduct of the election.

The President then called upon the Secretary to read an abstract of the report of the President to the Institute for the year 1930. This report was read and the Secretary stated that it would appear in full in the Year Book.

At eight-thirty the President introduced Mr. Igor I. Sikorsky, Vice-President of the Sikorsky Aviation Corporation, as the speaker of the evening, who talked on the subject "The Future of Large Aeroplanes."

The paper of the evening was one of the most interesting presented before the Institute in recent years. It dealt with the developments of the multiple-motored large plane, described its evolution, and predicted its future from the developments of the past. Mr. Sikorsky combined most delightfully the story of his personal experiences and achievements with the scientific principles and data underlying the development of the new machines. He described the possibilities of machines to carry thousands of passengers, on the one hand, and on the other hand, another type of machine arranged to maintain an internal pressure of a large fraction of the atmosphere, which machines, he predicted, would run comfortably and safely at elevations of from twenty-five to fifty miles from the earth and at speeds equal to, or exceeding, six hundred miles per hour.

The paper was greatly appreciated by an enthusiastic audience, which stayed long after the close of the meeting, asking interested and intelligent questions.

Mr. Hayward, as President, dismissed the meeting at nine-forty-six p.m., with an expression of appreciation from the Institute to Mr. Sikorsky for the pleasure which he had given to all.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 14, 1931.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 14, 1931.

MR. HENRY B. ALLEN *in the Chair.*

The following reports were presented for final action:

No. 2924: Mason Dehydration Process and Apparatus.

This report recommended the award of the John Price Wetherill Medal to Mr. Arthur J. Mason, of Homewood, Illinois, "In consideration of his devising a process for the artificial curing of alfalfa that forms a valuable contribution to the art of agriculture, and of its successful application that has resulted in the production of a concentrated food for farm animals that is of the utmost importance to our economic existence."

No. 2932 and No. 2933 } The Standing Committee on The Franklin Medal presented its report for final action making the following recommendations:

"That two Franklin Medals be awarded in 1931—one to

DOCTOR WILLIS R. WHITNEY, of Schenectady, New York, 'In recognition of his valuable contributions to industrial chemistry, and of his signal success as organizer and director of the greatly productive research laboratory of the General Electric Company, a success due in large part to his appreciation of the potential value of pure research in invention and industry, to his judgment of men and to his generosity in dealing with them';

And the other to

SIR JAMES HOPWOOD JEANS, of London, England, 'In recognition of his many fruitful contributions to mathematical physics, especially in the realms of the dynamical theory of gases and the theories of radiation, and of his challenging explanations of astronomical problems and his illuminating expositions of modern scientific ideas'."

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting Board of Managers, January 21, 1931.)

NON-RESIDENT LIFE.

DR. CHARLES A. STINE, Vice-President and Director, E. I. duPont deNemours and Company, Wilmington, Delaware.

RESIDENT.

MR. WILLIAM C. CUSHING, Engineer of Standards, The Pennsylvania Railroad. For mailing: Room 1658, 1617 Pennsylvania Boulevard, Philadelphia, Pa.

MR. HENRY C. L. MILLER, JR., Mechanical Engineer, Foster Wheeler Corporation, 2214 Packard Building, Philadelphia, Pa.

NON-RESIDENT.

MR. FREDERICK C. HOLTZ, Electrical Engineer, Sangamo Electric Company, Springfield, Illinois.

DR. FOSTER E. KLINGAMAN, Professor of Physics, Ursinus College. For mailing: 624 Main Street, Collegeville, Pa.

MR. PARRY H. MOON, Electrical Engineer, Massachusetts Institute of Technology, Cambridge, Mass.

NECROLOGY.

DR. HENRY LEFFMANN.

The Franklin Institute suffered a great loss in the death of Dr. Henry Leffmann which occurred, after a protracted illness, on December 25, 1930, in his eighty-fourth year.

Dr. Leffmann was born in Philadelphia, September 9, 1847. He joined The Franklin Institute in 1872 and to the time of his death remained an active member. He had been a member of the Committee on Science and the Arts during several terms, and also a member of the Committee on Library. During his lifetime he had generously established a fund for the binding of books, which is known as "The Henry Leffmann Fund." At his death, the Institute was made one of the three beneficiaries of a trust fund of fifty-eight thousand dollars, the income of which is to be used by the Institute for the purchase of books in physics and chemistry especially.

On Friday, January 16, a Memorial Service was held in the auditorium of the Wagner Institute, at which meeting addresses were made by representatives of the Jefferson Medical College, Wagner Free Institute of Science, The Franklin Institute, Central High School of Philadelphia, Woman's Medical College of Pennsylvania, Philadelphia College of Pharmacy and Science, Society for Ethical Culture and the Engineers Club of Philadelphia, in memory of Dr. Leffmann and his distinguished services.

The Secretary of The Franklin Institute read the following paper, as the tribute of appreciation and affection of the Institute for Dr. Leffmann:

HENRY LEFFMANN.
SEPTEMBER 9, 1847–DECEMBER 25, 1930.

DR. HENRY LEFFMANN.

As the representative of The Franklin Institute, I deem it a privilege to attempt to express the grateful appreciation of that Institute for Dr. Henry Leffmann's association with it and of his invaluable services to it, and to express also the admiration and affection which the members of The Franklin Institute knew for Dr. Leffmann as a discriminating and productive scholar and especially, as a man who in spite of unusual gifts and attainments, remained always approachable and stimulating and "who loved to do mercy and to help his fellow men."

Dr. Leffmann joined The Franklin Institute twelve days after the present speaker was born, and remained to his death a support and an ornament to that old society. During fifty-eight years he has aided in the operations of the Institute as an influential member of the Committee on Science and the Arts, having control of the awards of the several medals possessed by the Institute, and as one of the Committee on Library, entrusted with the government of the reading room and library. During the period of his scientific activities he published in the JOURNAL OF THE FRANKLIN INSTITUTE twenty articles on such widely distributed subjects as photography, photographic chemistry, water-purification, diamond mining, digestive ferments, water supply of ancient Jerusalem, and the diatoms of Agar-Agar. In 1913, the first book review by Dr. Leffmann appeared in the JOURNAL. Since that time he has written literally hundreds of reviews upon the most varied subjects. He has not hesitated to show up the ignorance or the pretense of those whose writings were under criticism. He has been acid when acidity was called forth by the charlatanism, or the arrogance, of the works considered. He was a little impatient of stupidity, and did not regard the unfortunate possession of that quality as a necessary excuse for slovenliness of expression, or for the publication of articles of little merit. But he always strove for justice, he sought fairness and he combined humor and good-nature in his condemnation of error at the same time that he knew sympathy for the errant. These characteristics were so recognized by authors that a favorable review bearing the initials 'H. L.' came to be looked upon as a great desideratum.

In May, 1930, The Franklin Institute honored itself by electing Dr. Leffmann to honorary membership, honoris causa. I had the pleasure of presenting him for the honor, "in recognition of valuable services to science in research, in teaching, as former Port Physician of the City of Philadelphia and as a discriminating but good-tempered critic." And I had a full recompense for all trouble taken in the merry twinkle of the eye and the audible chuckle of Dr. Leffmann as he murmured, "Yes, a discriminating but good-tempered critic!!"

As an investigator, administrator and teacher, Dr. Leffmann's interests seem to have been limited only by the boundaries of human knowledge. His writings covered the fields of language, history, philosophy, politics, human welfare, hygiene, food-adulteration, and pedagogy as well as the many technical phases of the physical and biological sciences. Whatever the subject of the moment's consideration, it was treated with a breadth of knowledge and a refinement of taste which characterized all of Dr. Leffmann's writings to an unusual degree. There seems to be no complete list of his published papers but there is

competent evidence to show that, in addition to the book reviews mentioned above, these papers exceeded five hundred in number. They displayed a surprising breadth of knowledge, combined with reliable accuracy of detail. They showed an unfailing courage and singleness of purpose. And they were permeated with a gentleness of humor which must have made criticism less unpalatable, and have prevented rancor.

Others will speak with greater competence than I of Dr. Leffmann's helpfulness to his associates and assistants. There had been no opportunity for me to observe him in relationship to others who were his assistants or associates in active investigations. When the privilege of association with him came, he had already passed far beyond the Psalmist's limitation of "three score years and ten." The gentleness of advancing years had come to him but with none of the evidences of age which often are distressing. Struck at once by the breadth of his knowledge and by the moderation of his judgment, I found him a sage advisor and a helpful friend. His friendly advice based upon his knowledge of things biological more than once deterred me, as Secretary of the Institute, from venturing into the to me strange world of biology. Time has confirmed his uttered judgments and has ever increased the measure of gratitude I feel towards him for his guidance.

I can speak with full knowledge of the nicety of feeling which he expressed, and of the consideration for the sensibilities and opinions of others always manifested by this great scholar. We all knew of his encyclopedic and accurate memory and did not hesitate to substitute an answer to a query addressed to him for a search in a work of reference. How much better was an answer enlivened by Dr. Leffmann's wit than any quotation from any product of the printer's art!

We all enjoyed the ebullitions of his quaint, quizzical humor and found it a joyous victory to be able to tell him a new story. We admired the simplicity and courage of the man. Life for all of us in The Franklin Institute was enriched and made happier by the privilege of association with this courteous gentleman, this erudite but stimulating scholar, this "Happy Warrior" in the world's combat against ignorance and prejudice.

If it were my privilege to write an epitaph to Dr. Leffmann dead, as it would have been a pleasure to pen a eulogy of him living, it would be as follows:

"Here lies a man, brave, simple, true;
Where'er he went high aspirations grew;
He's gone beyond, to play a nobler part,
The love of truth and knowledge filling all his heart."

Mr. H. N. Taylor, Philadelphia, Pa.
Mr J. T. Wilkin, Connersville, Ind.

CHANGES OF ADDRESS.

MR. WALTER ATLEE, 26 East Mount Vernon Place, Baltimore, Maryland.
MR. JOHN L. COX, 609 East Gravers Lane, Chestnut Hill, Philadelphia, Pa.
MR. W. W. FREEMAN, President, Intercontinents Power Company, 120 Wall Street, New York City.
MR. FRANCIS H. GILPIN, 30 Edgemont Ave., Summit, N. J.

MR. A. C. HOWARD, 74 Trinity Place, New York City.
MR. W. N. JENNINGS, 10 South Eighteenth Street, Philadelphia, Pa.
MR. DONALD L. KELLOGG, 18 Harper Street, Rochester, N. Y.
MR. HENRY W. LANDENBERGER, 6138 North Eleventh Street, Philadelphia, Pa.
MR. FREDERICK J. PEARSON, 1243 Merchandise Mart, Chicago, Illinois.
MR. WALTER PETERSON, 1514 Beacon Street, Brookline, Mass.
DR. GEORGE D. ROSENGARTEN, Royal Palm Dock, Miami, Florida.
MR. W. L. SAUNDERS, 2D, Aero Club of Pennsylvania, 1339 Walnut Street, Philadelphia, Pa.
MR. A. F. SHATTUCK, P. O. Box 237, Indio, California.
MR. A. KELVIN SMITH, Route 2, Box 286, Tucson, Arizona.
MR. CLINTON F. SNYDER, Washington-Lee Apartments Cumberland, Maryland.
DR. WILLIAM STERICKER, 540 Riverview Avenue, Swarthmore, Pa.
MR. JAMES STOKLEY, The University Club, Sixteenth and Locust Streets, Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

ADAM, NEIL KENSINGTON. The Physics and Chemistry of Surfaces. 1930.
American Annual of Photography, 1931. Volume 45. 1930.
American Institute of Chemical Engineers. Transactions. Volume 24. 1930.
American Institute of Electrical Engineers. Quarterly Transactions. Volume 49, Number 4. October 1930.
American Society for Testing Materials. A.S.T.M. Standards 1930. Pts. 1–2. Two volumes. 1930.
Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots and Reciprocals of All Integer Numbers up to 10,000. Third edition, edited by L. J. Comrie. 1930.
BRAUER, LUDOLPH. Albrecht Mendelssohn Bartholdy, and Adolf Meyer, editors. Forschungsinstitute: ihre Geschichte, Organisation, und Ziele. Zweiter Band. 1930.
BRONSON, BARNARD S. Nutrition and Food Chemistry. 1930.
Chemisches Zentralblatt. Generalregister über die Jahrgänge 1925–1929. Teil I, Autorenregister.
CROSBY, EVERETT U., HENRY A. FISKE AND H. WALTER FORSTER. Hand-Book of Fire Protection. Seventh edition. 1925.
ELLIS, DAVID, AND DUGALD CAMPBELL. The Science and Practice of Confectionery. 1928.
GRANVILLE, WILLIAM ANTHONY. Elements of the Differential and Integral Calculus. Revised by Percey F. Smith and William Raymond Longley. 1929.
GUNN, J. ALEXANDER. The Problem of Time. An Historical and Critical Study. 1930.
HARVEY, A. Laundry Chemistry. 1926.
LAVOISIER. Das Wasser. Übersetzt und herausgegeben von Dr. Finkelstein. Ostwald's Klassiker der exakten Wissenschaften. Nr. 230. 1930.
Louisiana Sugar Manual. Published by A. B. Gilmore. 23d annual edition. 1930.

MEYER, RUDOLF. Die Haloerscheinungen. Probleme der kosmischen Physik. Bd. XII. 1929.

SMYTH, HENRY FIELD, AND WALTER LORD OBOLD. Industrial Microbiology: the Utilization of Bacteria, Yeasts and Molds in Industrial Processes. 1930.

SWINTON, ALAN A. CAMPBELL. Autobiographical and Other Writings. 1930.

TIMOSHENKO, S. Strength of Materials. Two volumes. 1930.

Verein deutscher Ingenieure. Beiträge zur Geschichte der Technik und Industrie. Herausgegeben von Conrad Matschoss. Bd. 18–20. Three volumes. 1928–1930.

BOOK REVIEWS.

THE SCIENCE OF VOICE. A book on the singing and speaking voice based upon the latest research in physics and physiology, with advice to those interested in talking movies and other mechanical reproducing devices, by Douglas Stanley, M.S. (New York University), A. C. G. I. (London University). With a Preface by H. H. Sheldon, Ph.D. (Professor of Physics at New York University). VI–327 pages, 8vo. New York, Carl Fisher, Inc. n.d.

Scientific principles and investigative methods are applied to the art of singing. The reviewer had the great pleasure of hearing the author of this book, Douglas Stanley, clearly and forcefully present the results of his studies and researches in a lecture at The Franklin Institute during the Fall Series of 1930. Armed with experimentally determined facts on the physics of sound production and resonance as well as the physiological structure and mechanism of the human voice apparatus, Mr. Stanley holds up before the powerful, penetrating beam of his analytical X-ray, many common misconceptions regarding the manner in which human tones are produced, sustained and modulated. The result—many of these theories are found to be erected on false premises.

Mr. Stanley's studies have produced many interesting and somewhat unexpected results. He has reached the conclusion that the mouth of a singer does not form an ideal resonating cavity and that its importance as such has hitherto been greatly overestimated. Contrary to generally accepted practice the student should begin at the extremes of the upper and lower registers and work toward the middle of the range. The pianissimo tone should be one of the last phases of technic to be taught. There is no such thing as "placing the voice." Much valuable information has been gained by studying the rate of breath expulsion during the singing of the different tones.

With the advent of such appliances as the oscillograph, the harmonic analyzer, vacuum tube amplifiers, sound filters, high speed photography, etc., a new era in vocal training is predicted. Teachers will be required to have a thorough knowledge and understanding of physics and physiology. "Some day we will have the science of vocal technic standardized. . . . The teacher of voice will have to qualify, just as does the physician, before he is allowed to practise. When this day does at last dawn . . . we shall find that thousands of singers will be produced who can sing as well as the best Italians, are as musical as the German artists, and can act as well as the actors of the legitimate stage of America and England."

The book is divided into three sections. The first, "An Approach to the Subject through Physical Principles" has been written by Stanley S. A. Watkins, who deals with sound in the respect to its nature, quality and resonance, also vocal sounds, their recording and reproduction. Section two, on Vocal Technic, treats of the human voice, its producing apparatus and controlling mechanism, with a wealth of detail, clearness and simplicity in exposition. Section three is on Interpretation and Musicianship. The student will find this section very instructive and containing many valuable pointers for the would-be vocal artist.

T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS:

Report No. 365. Aerodynamic Characteristics of Circular-Arc Airfoils at High Speeds, by L. J. Briggs and H. L. Dryden. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

The aerodynamic characteristics of eight circular-arc airfoils at speeds of 0.5, 0.65, 0.8, 0.95, and 1.08 times the speed of sound have been determined in an open-jet air stream 2 inches in diameter, using models of 1-inch chord. The lower surface of each airfoil was plane; the upper surface was cylindrical. As compared with the measurements described in N. A. C. A. Technical Report No. 319, the circular-arc airfoils at speeds of 0.95 and 1.08 times the speed of sound are more efficient than airfoils of the R. A. F. or Clark Y families. At a speed of 0.5 times the speed of sound, the thick circular-arc sections are extremely inefficient, but thin sections compare favorably with those of the R. A. F. family. A moderate rounding of the sharp edges changes the characteristics very little and is in many instances beneficial. The results indicate that the section of the blades of propellers intended for use at high tip speeds should be of the circular-arc form for the outer part of the blade and should be changed gradually to the R. A. F. or Clark Y form as the hub is approached.

R.

PUBLICATIONS RECEIVED.

Principles of Engineering Thermodynamics, by Paul J. Kiefer, A.B., M.E. and Milton C. Stuart, M.E. 545 pages, illustrations, 8vo. New York, John Wiley & Sons, 1930. Price $4.50.

A Manual of Celestial Photography. Principles and practice for those interested in photographing the heavens, by Edward Skinner King, Sc.D., with a foreword by Harlow Shapley, Ph.D. 177 pages, illustrations, 12mo. Boston, Eastern Science Supply Company, 1931.

An Introduction to Structural Theory and Design Theory, by Hale Sutherland, A.B., S.B. and Harry Lake Bowman, S.M. 318 pages, tables, illustrations, 8vo. New York, John Wiley & Sons, Inc., 1930. Price $3.50.

The Dynamic Universe, by James Mackaye. 308 pages, 8vo. New York, Charles Scribner's Sons, 1931. Price $3.50.

Probabilities et Statistiques, par R. de Montessus de Ballore, preface de M.

Alliaume. 211 pages, illustrations, tables, quarto. Paris, Librairie Scientifique Hermann et Cie, 1931. Price 60 francs.

Cours de Physique, par M. A. Lafay, Tome 1, Acoustique-Electricite, Cours de L'Ecole Polytechnique. 666 pages, tables, illustrations, quarto. Paris, Gauthier-Villars et Cie, 1931. Price 125 francs.

Abridged Scientific Publications from the Kodak Research Laboratories, Volume XIII, 1929. 271 pages, tables, illustrations, 8vo. New York, Eastman Kodak Company, 1930.

Applications of Interferometry, by W. Ewart Williams, M.Sc., with a preface by O. W. Richardson, F.R.S. 104 pages, illustrations, 16mo. New York, E. P. Dutton & Co., Inc. No date.

National Advisory Committee for Aeronautics. Technical Notes, No. 359, A Balanced Diaphragm Type of Maximum Cylinder Pressure Indicator, by J. A. Spanogle and John H. Collins, Jr. Langley Memorial Aeronautical Laboratory, 7 pages, illustrations, quarto, Washington, Committee, 1930. No. 360, The Pressure Distribution Over a Square Wing Tip on a Biplane in Flight, by Richard V. Rhode and Eugene E. Lundquist, Langley Memorial Aeronautical Laboratory, 21 pages, tables, illustrations, quarto, Washington, Committee, 1931. Sixteenth Annual Report, 66 pages, plates, quarto, Washington, Committee, 1930.

Index to A. S. T. M. Standards and Tentative Standards, as of September 1930, 113 pages, 8vo. Philadelphia, American Society for Testing Materials, 1930.

U. S. Department of Commerce, Coast and Geodetic Survey. Results of Observations made at the United States Coast and Geodetic Survey Magnetic Observatory at Sitka, Alaska, in 1923 and 1924. 102 pages, quarto. Washington, Government Printing Office, 1930. Price 50 cents.

La Lumiere Monochromatique sa Production et son Emploi en Optique Pratique, par M. Ch. Fabry. 39 pages, illustrations, 8vo. Paris, Revue d'Optique theorique et instrumentale, 1930.

CURRENT TOPICS.

Elementary Principles of Wave Mechanics. E. C. KEMBLE. (*Phys. Rev. Supp.*, Vol. 1, No. 2.) The American Physical Society has placed all interested in physics under obligations by establishing this new periodical to provide "comprehensive, authoritative and timely discussions of the problems which are, at the present time, of especial interest and importance."

Professor Kemble's paper, which is continued in No. 3 of the Supplement, is necessarily largely garbed in mathematical language. There are, however, portions of general interest. Observation and experiment in the twentieth century have led to these important advances in physics: "(*a*) the discovery and exploitation of the corpuscular character of radiation; (*b*) the discovery and exploitation of the existence of discrete atomic and molecular energy levels, bringing into mechanics a new kind of atomicity superposed on the atomicity of electron and proton; (*c*) the discovery of the wave-like character of matter as exhibited in the now famous experiments of Davisson and Germer." In the development of theoretical physics the century has two periods. In the earlier, extending up to 1925, the leaders were Planck, Einstein, Bohr and Sommerfeld. The corpuscular theory of radiation arose and demonstrated its value by explaining the Compton effect. The wave theory, however, could not be discarded because it continued to furnish a simple explanation of such optical phenomena as interference and diffraction. At the same time Bohr's quantum theory of atomic structure provided remarkable explanations of line and band spectra. In the second period since 1925 there grew up matrix mechanics and the wave mechanics of de Broglie and Schrödinger. At first separate these are now combined in "Quantum Mechanics." In the form of quantum mechanics now most widely used, the dualistic nature of radiation is accepted as a brute fact to be described rather than explained or exercised. A similar dualistic nature is ascribed to matter and thus a unification in the treatment of matter and radiation is attained. The discovery of the fundamental similarity between matter and radiation is one of the most striking features of present physical theory. Differences, of course, remain and we can by no stretch of the imagination identify these two fundamental modes of existence, but the analogy is far-reaching enough to permit the use of observations regarding the characteristics of

radiation as guides in the formation of a theory of matter. "Thus the optical phenomena of regular reflection and of diffraction were true sign-posts pointing to similar effects with electrons. The dualistic nature is more readily seen in the case of radiation than with light. The wave nature of radiation is especially striking in the region of long wave-lengths while in the short lengths of the X-rays its corpuscular nature comes to the fore." As the transition from one part of the spectrum to the other is continuous, the dualism is inescapable. What is the upshot of the battle between the wave theory of light and the corpuscular theory? The answer is, "A deadlock." The wave theory gives a simple and accurate account of interference, diffraction and dispersion. The corpuscular theory gives a simple and accurate account of the fundamental laws of the photo-electric effect and the Compton effect. Neither point of view gives a satisfactory description of the whole field of optics." In a few instances of which the Doppler effect is one the two theories predict the same result and this is verified by experiment. In other cases part of an effect is explained by one theory and a second part by the other theory. Some combination of the two theories is clearly needed. It will not do to assume that light particles are merely small portions of space filled with waves, for such a conception fails to explain all the facts. "A more successful way of uniting the two theories is to adopt a frankly dualistic point of view by assuming that light consists of *both* spreading waves and corpuscles. If we postulate such a dualism, we can suppose that the electromagnetic waves of Maxwell's theory act as guides to the corpuscles and thus produce interference phenomena, while the localization of energy and momentum in the photons accounts for the photo-electric effect, etc."

In the early part of the mathematical development of the theory of quantum mechanics the author pauses to apply a formula to concrete bodies. He calculates that a golf ball moving a millimeter in 10 seconds and weighing 47 grams is accompanied by a wave disturbance having a wave-length of only 1.4×10^{-26} cm. For atoms and molecules the corresponding wave-lengths are much greater. "This means that diffraction effects are hopelessly beyond the reach of experiment in the case of large scale bodies. The wave-length of an oxygen molecule with a speed corresponding to the mean thermal energy at 300° K. is approximately 1.45×10^{-8} cm., while that for an electron with a 10 volt kinetic energy is 5.3×10^{-8} cm. As these wave-lengths are of the order of magnitude of atomic diameters, it is clear that diffraction effects must play a prominent part in atomic dynamical problems."

G. F. S.

Spectroscopy of Soft X-Rays. T. H. OSGOOD. (*Phys. Rev. Supp.*, Vol. 1, No. 2.) The last half century has witnessed the lengthening of the spectrum of radiation by the addition of long electrical waves to one end and of the very short X-rays and cosmic rays to the other. After the recognition of the wave nature of X-rays and of the extreme smallness of their wave-lengths a gap was left between the shortest waves in the ultra-violet and the longest in the X-ray region. Progress was made from both sides of the unknown area. Schumann, Lyman and Millikan worked out from the ultra-violet until the last named with his collaborators in 1921 extended the optical spectrum to a wave-length as small as 136 A. Employing photography to record X-rays obtained from crystal gratings, Siegbahn and Thoraeus reaching out for longer wave-lengths found on their plate in 1925 the imprint of a line corresponding to 25 A. This left a stretch of about 2½ octaves still unknown. Whether from the optical or the X-ray side experimentation was difficult because of the need of working in a high vacuum on account of the small penetration of the short waves concerned. Progress from the optical side was hampered by the small reflecting power of ruled gratings for short waves. To proceed from the X-ray side crystals having large grating spaces were called for and these were hard to get. Some information concerning the unexplored region was furnished by a study of photo-electric effects. This was, however, not very clear and a new technique was sought. Dauvillier in 1927 by methods now replaced by more accurate and easier ones succeeded in photographing a few lines of oxygen, carbon, boron and thorium between 20 A. and 120 A. He got the equivalent of a crystal by depositing a layer of melissic acid only a few hundred molecules thick upon a lead plate. The long-chain molecules set themselves in order like the pile of carpet and the grating space was about a dozen times as large as in commonly used crystals. His numerical results are not very reliable.

The essential feature of the new technique lay in letting the radiation to be resolved fall at angles of incidence greater than 80°, and commonly almost 90°, upon the optical grating mounted in the vacuum spectrograph. In 1923 A. H. Compton published experiments in which monochromatic X-rays of wave-length near 1 A. were reflected from a polished surface at an angle of incidence only a few minutes less than 90°. He found that there was a limiting angle at which the rays ceased to be reflected. Using different wave-lengths and different mirrors he drew the conclusions that the critical glancing angle increases with the wave-length and with the density of the substance of the mirror. The glancing angle is

the angle between the direction of the incident ray and the plane of the mirror. Puccianti of Pisa suggested the substitution of a ruled grating for the plane mirror on which the X-rays were incident. A. H. Compton and Doan in 1925 did this and got a diffraction spectrum of three orders from X-rays by the use of a speculum grating. Thibaud later showed that a glass grating is superior to a metal one, giving greater reflection and sharper lines.

A ray of visible light moving through water can be totally reflected at the surface between the liquid and air. This follows from the velocity of light being greater in air than in water. Similarly, from the total reflection of X-rays in air from metal or glass the conclusion is drawn that their velocity is greater in the solid than in air, that is, that their velocity in the glass and metal exceeds the velocity of light in air. The refractive index (air-solid) is a little less than unity. Its difference from unity is roughly in proportion to the square of the wave-length, so that for larger wave-lengths greater glancing angles can be used. In 1927 Osgood using a concave grating in America and Thibaud with a plane grating in Europe bridged the gap between the X-ray and the optical spectra.

As has been said, the refractive index of X-rays is less than one. For the ultra-violet rays it is considerably larger than unity. It is already known that with X-rays there occurs a phenomenon similar to anomalous dispersion in light. One of the problems to be solved by further investigation of the soft X-rays is the way in which the transition from the long optical waves to the short Röntgen rays takes place with the accompanying change in value of the index of refraction.

G. F. S.

The Symmetry of Time in Physics. GILBERT N. LEWIS. (*Science*, June 6, 1930.) In 1926 Professor Lewis advanced a theory of light that was in disaccord with the commonly held views of cause and effect. "Instead of assuming the time-honored unidirectional causality, in which cause inevitably precedes effect, it proved necessary to assume that the present phenomena of a system are determined no more by the past states of the system than by its future states. Several recent developments in physics make this assumption seem less startling now than then; indeed I am fully convinced that there is no other way in which we can account for the known phenomena of light."

The author ascribes the prevailing conception of time as flowing in one direction to the phenomena of memory and consciousness. "Was Newton right in deliberately introducing into physics this

common idea of the *flow* of time? Surely in one great branch of physics which we owe to his genius, the mechanics of conservative systems, it has long been recognized that there is need for nothing more than the simple idea of symmetrical time, which makes no distinction between past and future." The thesis is now defended that in all parts of physics and chemistry symmetrical time or time flowing both forward and backward suffice in the explanation of phenomena. To test the correctness of this hypothesis the various fields of physics are examined. "All the equations of mechanics are equally valid when t is replaced by $-t$. The chance of error is the same in calculating an eclipse of a thousand years ago or of a thousand years hence." A mythical precursor of Minkowski is made to say "Every equation and every explanation used in physics must be compatible with the symmetry of time. Thus we can no longer regard effect as subsequent to cause. If we think of the present as pushed into existence by the past, we must in precisely the same sense think of it as pulled into existence by the future." In the field of thermodynamics the conclusion is reached "that the irreversible process neither implies one-way time nor has any other temporal implications. Time is not one of the variables of pure thermodynamics." In optics the laws "are entirely symmetrical with respect to the emission and absorption of light. The whole science of optics leaves nothing to be desired with respect to symmetry in time. When time is considered reversed, the emitting and absorbing objects merely exchange rôles, but the optical laws remain unchanged." This conception leads to a way of regarding radiation in which "we must assign to the emitting and to the absorbing atom equal and coördinate rôles with respect to the act of transmission of light." In electromagnetics Maxwell's equations hold both for t and for $-t$, yet he obtained from them the old one-way theory of radiation. "In the whole history of physics this is the most remarkable example of the suppression by physicists of some of the consequences of their own equations, because they were not in accord with the old theory of unidirectional causality."

The author is convinced of the fruitfulness of his hypothesis of two-way time. "We have seen that if science long ago had accepted the principle of symmetry in time, it would have eliminated the idea of unidirectional causality which has led to so many of the errors of classical physics. From this principle could have been deduced the atomic structure of matter and the newer thermodynamics. By its aid the flaws in the older theories of radiation and in the electromagnetic theory would have been seen. Moreover the idea that light passes only from one particle to one particle, and that in this process the emitting and receiving atoms play coördinate parts, was

directly derivable from the law of the symmetry of time. It is remarkable that so many positive conclusions result from the negative statement that physics requires no one-way time, but more important conclusions have been derived from the similar negative statements that we cannot have a perpetual motion machine and that we cannot determine absolute velocities. Whether the new law will be successful in leading to new and unexpected conclusions remains to be seen."

G. F. S.

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